Sanghoon Baek (University of Ottawa)

Essential dimension of simple algebras and its application to algebraic groups of type $A_n$

In this talk, we introduce the notion of essential dimension of an algebraic structure and discuss some recent results on the essential dimension of certain classes of central simple algebras. We also relate these results to the essential dimension of split simple groups of type $A_n$.

Eva Bayer-Fluckiger (École Polytechnique Fédérale de Lausanne)

Isometries of quadratic spaces

Let $k$ be a field of characteristic not 2, and let $V$ be an inner product space over $k$. In 1969, Milnor raised the following question: which monic, irreducible polynomials can be minimal polynomials of isometries of $V$? The aim of this talk is to give a partial answer to this question, in particular when $k$ is an algebraic number field.

Jodi Black (Emory University)

Zero cycles on principal homogeneous spaces

The following is an open question of Jean-Pierre Serre. “Let $k$ be a field, let $G$ be a connected linear algebraic group over $k$, and let $X$ be a principal homogeneous space under $G$ over $k$. If $X$ admits a zero cycle of degree one, does $X$ have a $k$-rational point?” We give a positive answer to the question in certain settings.

Baptiste Calmès (Université d’Artois)

Cohomology theories on projective homogeneous varieties

I’ll explain how to compute cohomology groups of projective homogeneous varieties for various types of cohomology theories: cobordism, Witt groups, etc.

Vladimir Chernousov (University of Alberta)

On conjugacy of Cartan subalgebras in extended affine Lie algebras

One of the central theorems of classical Lie theory is that all split Cartan subalgebras of a finite dimensional simple Lie algebra over an algebraically closed field are conjugate, a theorem of Chevalley. This result yields the most elegant proof that the type of the root system of a simple Lie algebra is its invariant. In infinite dimensional Lie theory maximal abelian diagonalizable subalgebras (MADs) plays the role which Cartan subalgebras plays in the classical theory. In the talk we address to the problem of conjugacy of MADs in a big class of Lie algebras which are called in the literature by extended affine Lie algebras (EALA). To attack this problem we develop a bridge which connects the world of MADs in infinite dimensional Lie algebras and world of torsors over the Laurent polynomial rings.

Jean-Louis Colliot-Thélène (Université Paris-Sud XI, CNRS)

Unramified cohomology

This is a survey talk on unramified cohomology groups of function fields. A classical example of such a group is the Brauer group of a smooth projective variety. Motivation for the study of these groups comes from at least two directions:

1. These groups are birational invariants and are essentially trivial when computed on a retract rational variety. They may thus detect nonrationality.
2. These groups may yield information on the Chow groups of algebraic cycles.

Charles De Clercq (Université Pierre et Marie Curie Paris VI)

Motivic indecomposable summands of projective $PGL_1(A)$-homogeneous varieties

Motivic decompositions of projective homogeneous varieties and their relations with classical discrete invariants have been intensively studied recently. We will consider the case of adjoint absolutely simple algebraic groups of inner type $A_n$, that is to say $PGL_1(A)$-homogeneous varieties for some central simple algebra $A$. We give a purely algebraic classification of indecomposable direct summands arising in such varieties in the category of Grothendieck Chow motives with coefficients in $F_p$, and discuss their behaviour when extending...
the ring of coefficients.

Andrew Dolphin (University of Konstanz)
"Decomposition of algebras in characteristic 2"
In this talk we present a decomposition theorem extending the usual Witt decomposition for hermitian forms over fields of characteristic 2. We will discuss an application of this decomposition to algebras with involution over fields of characteristic 2 in obtaining results on the effect of passing to a generic splitting field of the algebra on the isotropy of the involution.

Jérôme Ducoat (Institut Fourier, Université Grenoble I)
"Cohomological invariants of finite Coxeter groups"
In this talk, I will present a generalization of a Serre theorem for cohomological invariants of the symmetric group, stating that for every base field of characteristic different from 2, calculate an invariant on multiquadratic étale algebras determines completely this invariant. In the case of finite Coxeter groups, we are working with zero characteristic fields and we have to replace multiquadratic algebras by all cohomology classes coming from abelian subgroups generated by reflections. I will then determine as an application all invariants of some Weyl groups of type $B_n$.

Alexander Duncan (University of British Columbia)
"Versal Torsors"
Given an algebraic group $G$, a versal torsor for $G$ is a $G$-torsor from which any other $G$-torsor (over a field) can be obtained by pullback. I will discuss their importance for Galois cohomology and for computing essential dimension, and outline how they can be studied using birational and arithmetic geometry.

Eli Geva (Technion-Israel Institute of Technology)
"The cyclicity question for the projective Schur group"
Is any projective Schur algebra Brauer equivalent to a product of cyclics?

Philippe Gille (École Normale Supérieure)
"Torsors over Laurent polynomial rings and extended affine Lie algebras"
Our goal is to classify twisted toroidal Lie algebras namely Lie algebras which look like a simple complex Lie algebra $\mathfrak{g}$ extended to a Laurent polynomial ring $R$. We shall discuss mainly the case of the Lie algebra $\mathfrak{g} = \mathfrak{sl}(n)$ which relates to projective representations of finite abelian groups, quantum tori and Brauer groups of henselian fields. This is a report on joint work with Arturo Pianzola.

Stefan Gille (University of München)
"Chow motives of surfaces"
I will discuss some recent results on Chow-motives (with integral or finite coefficients) of surfaces. In particular I will introduce the upper motive of a geometrically rational surface which turns out to be a birational invariant.

David Grimm (University of Konstanz)
"Sums of squares in function fields in one variable"
We address current research on sums of squares in function fields over hereditarily pythagorean fields. Using recently discovered local-global principles based on field patching techniques, we determine the sums of squares structures in function fields of curves over $R((t))$.

David Harbater (University of Pennsylvania)
"Saltman's generic Galois extensions and problems in field theory"
In an influential 1982 paper, David Saltman introduced the notion of generic Galois extensions, and explained the relationship to Noether’s problem and to the exceptional case of the Grunwald-Wang theorem. Much related work on these topics was inspired by this paper, as was other work in field theory, especially on the inverse Galois problem and the arithmetic lifting problem. This talk will survey these ideas and their relationships.

Julia Hartmann (RWTH Aachen University)
"Local-global principles for torsors over arithmetic curves"
Patching methods are usually used to construct a global object from local data. When the machinery is applied in the converse direction, it leads to local-global principles. This talk explains such principles for torsors under certain linear algebraic groups and describes the kernel of the local-global map, which can be seen as an analogue of the Tate-Shafarevich group. Some of the results yield information about more classical local-global principles with respect to discrete valuations. (Joint work with David Harbater and Daniel Krashen.)

Brendan Hassett (Rice University)
Families of quartic del Pezzo surfaces
We discuss the structure and classification of families of quartic del Pezzo surfaces over the projective line, focusing on the geometry of their spaces of sections. Our motivation comes from arithmetic questions for function fields of curves over finite fields.

Hilaf Hasson (University of Pennsylvania)
Questions concerning the regular realizability of groups over the projective line over various fields.
Regular realizability of groups over the projective line over various fields is closely related to the inverse Galois problem. I will explain how. I will (briefly!) present several results and questions regarding the relationship between the “field of moduli” of a cover, the rationality of the branch locus, and the groups that may appear.

Olivier Haution (University of Nottingham)
Integrality of the Chern character and Steenrod operations for Chow groups
Steenrod operations are useful tools for the study of rationality properties of projective homogeneous varieties. The known constructions of these operations require to make some assumption on the characteristic of the base field. We discuss an integrality property of the Chern character with values in Chow groups, and deduce the existence of a weak form of some Steenrod operations, over an arbitrary field.

Detlev Hoffmann (Technische Universität Dortmund)
Witt kernels — a survey
The Witt kernel of a field extension is the kernel of the restriction map from the Witt ring or Witt group of a field to that of the field extension. It has been studied extensively for various types of field extensions, but many questions still remain open. We give a survey on the problem of determining Witt kernels, including fields of characteristic 2 where we consider the case of quadratic forms as well as the case of bilinear forms. We also mention results concerning Witt kernels for the graded Witt ring/group.

Yong Hu (Université Paris-Sud XI Orsay)
Quadratic forms over fraction fields of two-dimensional henselian domains
Let $K$ be either the function field of a $p$-adic curve or the fraction field of a two-dimensional henselian excellent local domain with finite residue field. In either case, $K$ may be realized as the function field of a regular excellent surface whose Brauer group is trivial. When $K$ is a $p$-adic function field, it is known that every quadratic form of rank at least 9 over $K$ is isotropic (Parimala and Suresh) and that every quadratic form of rank at least 3 satisfies the local-global principle with respect to discrete valuations of $K$ (Colliot-Thélène, Parimala and Suresh). In this short talk I will discuss what can be said in the latter case, i.e., when $K$ is the fraction field of a two-dimensional henselian domain with finite residue field.

Nikita Karpenko (Université Pierre et Marie Curie Paris VI)
Incompressibility of generic orthogonal grassmannians
Given a non-degenerate quadratic form over a field such that its maximal orthogonal grassmannian is 2-incompressible (a condition satisfied for generic quadratic forms of arbitrary dimension), we apply the theory of upper motives to show that all other orthogonal grassmannians of this quadratic form are 2-incompressible. This computes the canonical 2-dimension of any projective homogeneous variety (i.e., orthogonal flag variety) associated to the quadratic form. Moreover, we show that the Chow motives with coefficients in the field of 2 elements (and therefore also in any field of characteristic 2) of those grassmannians are indecomposable. That is quite unexpected, especially after a recent result on decomposability of the motives of incompressible twisted grassmannians.

Max-Albert Knus (ETH Zürich)
Triality, from projective geometry to cohomology
The first part gives some historical indications about triality, in the second part new results (Chernousov, Tignol, K.) about the classification of triality up to conjugacy are presented and the last part is devoted to the contributions of David Saltman to triality.

Boris Kunyavskii (BAR-ILAN UNIVERSITY)

*Equations in simple algebras*

Given an element \( P(X_1, ..., X_d) \) of the free Lie \( K \)-algebra, for any Lie algebra \( g \) we can consider the induced polynomial map \( P : g^d \to g \). Assuming that \( K \) is an arbitrary field of characteristic different from 2, we prove that if \( P \) is not an identity in \( sl(2, K) \), then this map is dominant for any Chevalley algebra \( g \). This result can be viewed as a weak infinitesimal counterpart of Borel’s theorem on the dominancy of the word map on connected semisimple algebraic groups. As in the group case, the proof is based on a construction of division subalgebras due to Deligne and Sullivan. We also prove that for the Engel monomials \([[[X,Y], Y], ..., Y]\) and, more generally, for their linear combinations, the map \( P \) is, moreover, surjective onto the set of noncentral elements of \( g \) provided that the ground field \( K \) is big enough, and show that for monomials of large degree the image of this map contains no nonzero central elements. We also discuss consequences of these results for polynomial maps of associative matrix algebras.

Yongqi Liang (UNIVERSITÉ PARIS-SUD XI ORSAY)

*Br"{a}uer-Manin obstruction for rational points and for 0-cycles.*

We would like to indicate some relations between the question of Br"{a}uer-Manin obstruction for rational points and that for 0-cycles.

Max Lieblich (UNIVERSITY OF WASHINGTON)

*Saltman’s meteorology* 

I will explain how to interpret Saltman’s description of singular points in the ramification locus using Br"{a}uer groups of stacks and give some applications to classical problems.

Roland L"{o}tscher (UNIVERSIT"{A}T M"{U}NCHEN)

*Essential dimension of separable subalgebras*

Let \( A \) be a central simple algebra and \( E \) a separable subalgebra of \( A \). In this talk I will discuss the essential dimension of the normalizer \( G \) of the group scheme \( GL(1, E) \) of invertible elements of \( E \) in \( GL(1, A) \), focusing on the two extreme cases in which \( A \) is either split or a division algebra. In the division algebra case I will show how to compute the exact value of the essential dimension of \( G \), assuming only that the degree of \( A \) is a prime power. Surprisingly the split case turns out to be much harder. I will explain how it is linked to classical problems in essential dimension, like computing the essential dimension of central simple algebras of fixed degree or étale algebras of fixed dimension.

Mark MacDonald (UNIVERSITY OF BRITISH COLUMBIA)

*Essential p-dimension of normalizers of maximal tori*

I will explain a technique for finding the exact value of the essential \( p \)-dimension of the normalizer of a split maximal torus inside most connected simple linear algebraic groups. These values give new upper bounds on the essential \( p \)-dimension of some simple groups, including some exceptional groups.

Eliyahu Matzri (TECHNION-ISRAEL INSTITUTE OF TECHNOLOGY)

*Non cyclic algebras with n-central elements.*

Let \( D \) be a division algebra (i.e. every non zero element of \( D \) is invertible) of finite dimension over it’s center, \( F \). It is known that \([D : F] = n^2\) for some natural number \( n \) which is called the degree of \( D \). For every element \( \alpha \) in \( D \), \( F[\alpha] \) is a field. We call \( D \) a cyclic algebra if it has a subfield of dimension \( n \) which is Galois over \( F \) with group \( C_n \). More generally we call \( D \) a crossed product with respect to a group \( G \) over \( F \) with group \( G \). Cyclic algebras are very important in the theory of finite dimensional division algebras due to a wonderful theorem of Merkurjev and Suslin stating that if the center contains enough roots of unity \( M_k(D) \) for some natural \( k \) is isomorphic to the tensor product of cyclic algebras. For \( D \) of degree a prime \( p \), it is a well known theorem of Albert stating that \( D \) is cyclic iff it contains a non central element \( d \) whose \( p \)-th power is in the center. This theorem is clear if \( F \) contains a \( p \)-th root of unity but is true even if it does not. A natural question is, does Albert’s criterion hold for algebras of prime power degree. Albert gave a counter example to his criterion in degree four—that is he gave a degree four division algebra with a 4-central element which is not cyclic. In a recent work with L. H. Rowen and U. Vishne we give examples of algebras \( D \), of degree \( p^2 \) and a non central element whose \( p^2 \) power is in the center but \( D \) is not cyclic thus showing
Alberts criterion for cyclicity is false for algebras of degree $p^2$ for every prime $p$.

**Kelly McKinnie (University of Montana)**

*Lifting Brauer classes to function fields of $p$-adic curves.*

Let $X$ be a smooth integral curve over $\mathbb{Z}_p$ and let $K(X)$ be the completion of the function field of $X$ at the special fiber. This talk will describe joint work with E. Brussel and E. Tengan which takes a Brauer class over $K(X)$ (of index prime to $p$) and lifts it to a Brauer class over $K(X)$. As a consequence we lift indecomposable and non-crossed product division algebras over $K(X)$ to $K(X)$.

**Alexander Merkurjev (University of California Los Angeles)**

*Essential dimension in algebra*

Essential dimension of an algebraic object is the smallest number of algebraically independent parameters required to define the object. This notion was introduced by Buhler and Reichstein. Recent results, methods and ideas will be discussed.

**Michael Natapov (Technion-Israel Institute of Technology)**

*A graph theoretic approach to graded identities*

We consider the algebra $M_k(C)$ of $k$-by-$k$ matrices over the complex numbers and view it as a crossed product with a group $G$ of order $k$ by imbedding $G$ in the symmetric group $S_k$ via the regular representation and imbedding $S_k$ in $M_k(C)$ in the usual way. This induces a natural $G$-grading on $M_k(C)$ which we call a crossed product grading. We study the graded polynomial identities for $M_k(C)$ equipped with a crossed product grading. To each multilinear monomial in the free graded algebra we associate a directed labeled graph. This approach allows us to give new easy proofs of some known results in the PI theory. Our most substantial new result is the determination of the asymptotic formula for the $G$-graded codimension of $M_k(C)$.

**James O’Shea (University College Dublin / Universität Konstanz)**

*Isotropy over function fields of Pfister forms*

We will address the question as to which anisotropic quadratic forms become isotropic when extended to the function field of a Pfister form, discussing various aspects of this problem and outlining some relevant progress.

**Alena Pirutka (Université Paris-Sud XI, Ecole normale supérieure)**

*Unramified cohomology and Chow groups*

We will explain how one can relate the degree three unramified cohomology and the Chow group of codimension two cycles. Using this, we will exhibit an example of a smooth projective geometrically rational variety $X$ defined over a finite field $\mathbb{F}_p$ such that the map $CH^2(X)$ to $CH^2(\overline{X})^G$ is not surjective.

**Anne Queguiner-Mathieu (Université Paris Est Créteil - Université Paris 13)**

*J-invariant and triality*

The $J$-invariant, initially defined by Vishik for quadratic forms, has been extended to the more general setting of an inner algebraic group by Petrov, Semenov, and Zainoulline. Using a relation with indices of Tits algebras, one can compute it for algebras of small degree with orthogonal involution. The most interesting case is the degree 8 case, where the underlying automorphism group has type $D_4$. From this computation, and a classical result of representation theory, we get new restrictions on the possible values for the $J$-invariant, which were not obtained using the Steenrod operation.

**Mélanie Raczk (Université catholique de Louvain)**

*The 3-Pfister number of quadratic forms*

Let $F$ be a field of characteristic different from 2 containing a square root of $-1$. The 3-Pfister number of a quadratic form $q$ in the third power of the fundamental ideal of $F$, is the least number of terms needed to write $q$ as a sum of 3-fold Pfister forms. We use a combinatorial analogue of the Witt ring of $F$ to prove that, if $F$ is a 2-henselian valued field with at most two square classes in the residue field, then the 3-Pfister number of a $d$-dimensional quadratic form is less than or equal to $(d^2)/8$.

**Andrei Rapinchuk (University of Virginia)**

*On division algebras having the same maximal subfields*

The talk will be built around the following question: let $D_1$ and $D_2$ be two central quaternion division algebras...
over the same field $K$; when does the fact that $D_1$ and $D_2$ have the same maximal subfields imply that $D_1$ and $D_2$ are actually isomorphic over $K$? I will discuss the motivation for this question that comes from the joint work with G. Prasad on length-commensurable locally symmetric spaces, and will then talk about some available results. One of the results (joint with I. Rapinchuk) states that if the answer to the above question is positive over a field $K$ (of characteristic not 2) then it is also positive over any finitely generated purely transcendental extension of $K$. I will also discuss some generalizations to algebras of degree $>2$, and some recent qualitative (finiteness) results.

**Louis Rowen (Bar-Ilan University)**

*An algebraic approach to tropical mathematics*

The rapidly developing topic called “tropical mathematics,” has been based on two main approaches. Primarily, tropical curves have been defined as domains of non-differentiability of polynomials over the max-plus algebra, and also tropical mathematics has been viewed in terms of valuation theory applied to curves over Puiseux series. Unfortunately, semirings such as the max-plus algebra possess a limited algebraic structure theory, and also do not reflect these valuation-theoretic properties, thereby forcing researchers to turn to combinatoric arguments.

The object of this talk is to present an algebraic structure more compatible with algebraic structure theory and valuation theory than the max-plus algebra. We present a “layered” structure, “sorted” by a semiring which permits varying ghost layers, and indicate how it permits a direct algebraic description of tropical varieties. We also discuss factorization of polynomials, linear algebra, properties of the resultant, and multiple roots of polynomials. Explicit examples and comparisons are given for various “sorting” semirings such as the natural numbers and the positive rational numbers, and we consider how this theory relates to some recent developments in the tropical literature such as “characteristic 1,” “analytification,” and “hyperfields.”

**Nikita Semenov (Universität Mainz)**

*Shells of twisted flag varieties and non-decomposibility of the Rost invariant*

We relate the rationality of parabolic subgroups of algebraic groups and the Rost invariant. For groups of type $E_7$ the existence of such relation was conjectured by Markus Rost. This is a joint work with Victor Petrov and Skip Garibaldi.

**Jack Sonn (Technion-Israel Institute of Technology)**

*On the minimal ramification problem for semiabelian groups*

It is known (Kisilevsky and Sonn) that for any prime $p$ and any finite semiabelian $p$-group $G$, there exists a (tame) realization of $G$ as a Galois group over the rationals $\mathbb{Q}$ with exactly $d = d(G)$ ramified primes, where $d(G)$ is the minimal number of generators of $G$, which solves the minimal ramification problem for finite semiabelian $p$-groups. We generalize this result to obtain a theorem on finite semiabelian groups and derive the solution to the minimal ramification problem for a certain family of semiabelian groups that includes all finite nilpotent semiabelian groups $G$. Finally, we give some indication of the depth of the minimal ramification problem for semiabelian groups not covered by our theorem. (Joint with Hershy Kisilevsky and Danny Neftin)

**James Stankewicz (University of Georgia)**

*Rational points on Atkin-Lehner twists of Shimura Curves at ramified primes*

Determining the rational points of Atkin-Lehner twists of Shimura curves is of interest both in the study of $\mathbb{Q}$-curves and $\mathbb{Q}$-surfaces as well as finding Galois covers via Shih’s construction. However, even the determination of local points is a nontrivial problem. In this talk we’ll discuss how to determine whether or not there are local points at ramified primes, where the solution can be explained without (much) arithmetic of quaternion orders and ideals.

**V. Suresh (University of Hyderabad / Emory University)**

*Division algebras over function fields of surfaces d’après Saltman*

We discuss two central results on division algebras over function fields of surfaces due to Saltman. Let $D$ be a central division algebra of degree $n$ over the function field of a $p$-adic curve. Suppose that $n$ is coprime with $p$. The first result says that the degree of $D$ divides the square of the period of $D$. The second result says that if the degree of $D$ is a prime, then $D$ is a cyclic algebra.

**Jean-Pierre Tignol (Université Catholique de Louvain)**

*Total ramification*

Brussel’s description of the Brauer group of inertially closed graded fields and Henselian valued fields is related
to the canonical alternating form on the value group of totally ramified (graded) division algebras. (Joint work with Adrian Wadsworth)

Anthony Várilly-Alvarado (RICE UNIVERSITY)

Transcendental obstructions to weak approximation on general K3 surfaces

It is well-known that K3 surfaces over number fields need not satisfy the Hasse principle or weak approximation. All known counter-examples to date, however, involve K3 surfaces that are endowed with an elliptic fibration structure; in fact, the fibration is essential to the computation of Brauer classes that reveal obstructions to the Hasse principle and weak approximation. General K3 surfaces, i.e., K3 surfaces with geometric Picard rank one, do not enjoy this kind of structure. I will explain how to construct certain K3 surfaces of geometric Picard rank one, together with a transcendental quaternion algebra that obstructs weak approximation of rational points. This is joint work with Brendan Hassett and Patrick Varilly.

Bianca Viray (BROWN UNIVERSITY)

Descent on elliptic surfaces and transcendental Brauer elements

Transcendental elements in the Brauer group are notoriously difficult to compute. Wittenberg and Ieronymou have worked out explicit representatives for 2-torsion elements of elliptic surfaces, in the case that the Jacobian fibration has rational 2-torsion. We use ideas from descent to develop techniques to study the 2-torsion elements of elliptic surfaces without an assumption on the 2-torsion.

Alexander Vishik (UNIVERSITY OF NOTTINGHAM)

Motivic cohomology of quadrics

Considered as an object of triangulated category of Voevodsky, the motive of a quadric can be presented as an extension of simple motives $\mathcal{X}_Q, \mathcal{X}_Q^1, \ldots, \mathcal{X}_Q^d$, where $Q'$ is the Grassmannian of $i$-dimensional projective planes on $Q$, and $\mathcal{X}_S$ is the motive of the Chéch simplicial scheme associated with the pair $S \to \text{Spec}(k)$. The motive $\mathcal{X}_Q$ is a “form” of Tate-motive: over $\overline{k}$ it becomes isomorphic to the motive of a point. But over the ground field it is infinite-dimensional. And the degree of ”infiniteness” of this motive can be estimated by the ”rate of growth” of its motivic cohomology groups. It appears that the rate of growth of motivic cohomology of $\mathcal{X}_Q, \mathcal{X}_Q^1, \ldots, \mathcal{X}_Q^d$ carries the same information as the Elementary Discrete Invariant $EDI(Q)$ of our quadric (an invariant defined in terms of Chow groups of quadratic Grassmannians). This provides a new point of view on $EDI(Q)$. An interaction between these two invariants gives interesting results about both. It also suggests an approach to the Elementary Discrete Invariant for the algebraic groups of other types.

John Voight (UNIVERSITY OF VERMONT)

Quaternion rings and ternary quadratic forms

We consider the class of algebras of rank 4 equipped with a standard involution over an arbitrary commutative base ring. In particular, we characterize quaternion rings, those algebras defined by the construction of the even Clifford algebra of a (possibly degenerate) ternary quadratic form. This generalizes work of Gross and Lucianovic. Time permitting, we describe some consequences for a "Brauer monoid" of a commutative ring.

Adrian Wadsworth (UNIVERSITY OF CALIFORNIA SAN DIEGO)

$SK_1$ for valued division algebras, graded division algebras, and twisted rational division algebras

(This is joint work with R. Hazrat.) For any valuation $v$ on a division algebra $D$ finite-dimensional over its center $F$, there is an associated graded division algebra $gr(D)$, which carries much information about $D$ but is often easier to work with than $D$ itself. $gr(D)$ has a ring of quotients $q(gr(D))$ which is a division algebra finite-dimensional over the field $q(gr(F))$.

**Theorem:** If $v$ on $F$ is Henselian and $D$ is tame over $F$, then $SK_1(D)$ is isomorphic to $SK_1(gr(D))$, which is isomorphic to $SK_1(q(gr(D)))$.

This unifies many previous calculations of $SK_1$. Computation of $SK_1$ is significantly easier in the graded setting than for valued division algebras or twisted rational division algebras.

Kirill Zainoulline (UNIVERSITY OF OTTAWA)

The Grothendieck gamma filtration and the Rost invariant

Let $X$ be the variety of Borel subgroups of a simple and strongly inner linear algebraic group $G$ over a field $k$. We prove that the torsion part of the second quotient of Grothendieck’s gamma-filtration on $X$ is a cyclic group of order the Dynkin index of $G$. As a byproduct of the proof we obtain an explicit cycle that generates
this cyclic group; we provide an upper bound for the torsion of the Chow group of codimension-3 cycles on $X$; we relate the generating cycle with the Rost invariant and the torsion of the respective generalized Rost motives; we use this cycle to obtain a uniform lower bound for the essential dimension of (almost) all simple linear algebraic groups.

Maksim Zhykhovich (Université Pierre et Marie Curie Paris VI)

Motivic decomposability of generalized Severi-Brauer varieties

Let $F$ be an arbitrary field. Let $p$ be a positive prime number and $D$ a central division $F$-algebra of degree $p^n$, with $n > 0$. Let $m$ be an integer from the interval $[0, n - 1]$. We note by $M(SB(p^m, D))$ the Chow motive with coefficients in $F_p$ of the generalized Severi-Brauer variety $SB(p^m, D)$. It was proven by Nikita Karpenko that this motive is indecomposable for any prime $p$ and $m = 0$ and for $p = 2$, $m = 1$. We prove decomposability of $M(SB(p^m, D))$ in all the other cases. We also describe the complete motivic decomposition of $SB(p^m, D)$ for $p = 2$, $m = 2$, $n = 3$ and for $p = 3$, $m = 1$, $n = 2$. 

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