GROUP FERMAT: PROBLEM SOLVING STRATEGIES

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1. Strategy

What do you do when you get a problem that looks intimidating, with really big numbers or an overwhelming number of cases to try?

Start by trying some easier versions of the problem, or doing a few examples. Then look for patterns and generalize!

2. Problems

(1) The sum of six consecutive integers could be

A) 81       B) 88       C) 92       D) 98

(2) $3^{2013} - 3^{2012} =$

A) $3^1$       B) $3^{2011}$       C) $2 \times 3^{2012}$       D) $6^{1006}$

(3) If I multiply all whole numbers from 1 through 100, the largest power of 4 that is a factor of the product is

A) $4^{25}$   B) $4^{32}$   C) $4^{48}$   D) $4^{50}$

(4) $\frac{1}{3} \times \frac{2}{4} \times \frac{3}{5} \times \frac{4}{6} \times \cdots \times \frac{97}{99} \times \frac{98}{100} = \frac{1}{100} \times ?$

A) $\frac{3}{199}$       B) $\frac{2}{99}$       C) $\frac{1}{99}$       D) $\frac{2}{9900}$

(5) I wrote a list of consecutive positive integers beginning with 1. I then removed all multiples of 4, and I had 2345 integers left. What was the largest integer on my list after the numbers were removed?
A) 3126  B) 3127  C) 3129  D) 3130

(6) Which of the following is a sum of two prime numbers?
A) 31  B) 19  C) 23  D) 11

(7) The sum of two different odd numbers and an even number could be
A) 52  B) 61  C) 65  D) 77

(8) How many of the first 1000 positive integers are multiples of both 4 and 5 but not of 6?
A) 34  B) 42  C) 59  D) 58

(9) The hundreds digit of the product $123456789 \times 234567890$ is
A) 0  B) 1  C) 2  D) 3

(10) Challenge: On a blackboard are written the numbers 1 through 100. At every stage, two are selected, erased from the board, and their sum plus product is added to the list on the board. At any stage, you’re free to choose any two numbers. When the board is reduced to a single number, what possible values can it have?