LOGIC PUZZLES

1. WARM UP: THINK OUTSIDE THE BOX

(1) Twenty-four red socks and 24 blue socks are lying in a drawer in a dark room. What is the minimum number of socks I must take out of the drawer which will guarantee that I have at least two socks of the same color?

(2) A train leaves from Boston to New York. An hour later, a train leaves from New York to Boston. The two trains are going at exactly the same speed. Which train will be nearer to Boston when they meet?

(3) What happens if an irresistible cannonball hits an immovable post? That is, a cannonball that knocks over everything in its way meets a post which cannot be knocked over by anything.

(4) A man owned no watch, but he had an accurate clock which, however, he sometimes forgot to wind. Once when this happened he went to the house of a friend, passed the evening with him, went back home, and set his clock. How could he do this without knowing beforehand the length of the trip?

(5) A man is looking at a portrait. Someone asked him, “Whose picture are you looking at?” He replied: “Brothers and sisters I have none, but this man’s father is my father’s son.” (This man’s father means the father of the man who is in the picture). Whose picture was the man looking at?

2. LOGIC OF “ALL x ARE y”

For each of the following, draw a diagram to illustrate the assumptions. Determine whether the conclusion is true or false. If the conclusion is true, draw a proof tree showing that it is true.

Rules:

\[
\begin{align*}
\text{All } x \text{ are } x \\
\text{All } x \text{ are } y & \quad \text{All } y \text{ are } z \\
\text{All } x \text{ are } z
\end{align*}
\]

(1) Assume: All dogs are animals, All cats are animals
Conclusion: All dogs are cats.

(2) Assume: All animals are loyal, All dogs are animals
Conclusion: All dogs are loyal.
(3) Assume: All donuts are foods, All foods are sweet
   Conclusion: All foods are donuts.

(4) Assume: All pears are fruits, All apples are fruits
   Conclusion: All pears are pears

(5) Assume: All backpacks are bags, All bags are heavy
   Conclusion: All backpacks are heavy.

(6) Assume: All pigs are cows, All cows are fruits, All fruits are sour.
   Conclusion: All pigs are sour

(7) Assume: All sodas are cokes, All cokes are sprites
   Conclusion: All sprites are sodas

3. All $x$ are $y$, Some $x$ are $y$, No $x$ are $y$

For each of the following, determine whether the conclusion is true or false. If the conclusion is true, draw a proof tree showing that it is true. If the conclusion is false, make a model showing it is false.

Rules: Projected on the screen

(1) Assume: All notes are post-its, Some notes are notes
   Conclusion: Some notes are post-its

(2) Assume: All $n$ are $p$, Some $n$ are $n$, All $n$ are $q$
   Conclusion: Some $p$ are $q$

(3) Assume: No doctors are doctors
   Conclusion: All doctors are chefs

(4) Assume No sandwiches are candy bars
   Conclusion: No candy bars are sandwiches

(5) Assume: All $p$ are $q$, No $r$ are $q$, All $q$ are $w$
   Conclusion: Some $r$ are $p$

(6) Assume: Some poisons are pills, All pills are purple
   Conclusion: All poisons are purple

(7) Assume: All bags are pockets, No pouches are pockets
   Conclusion: No pouches are bags

(8) Assume: All bags are pockets, No pouches are pockets
   Conclusion: No bags are pouches
(9) Assume: Some pigs are predators, No predators are pets
   Conclusion: No pigs are pets

(10) Assume: Some maggots are flies, No flies are welcome
     Conclusion: No maggots are welcome

(11) Assume: Some doctors are fools, All fools are rich
     Conclusion: Some doctors are rich

(12) Assume: Some parrots are cars, All parrots are fish
     Conclusion: Some cars are fish

(13) Assume: No dogs are dogs, No cats are frogs
     Conclusion: No cats are dogs

4. Challenge Problems

(1) You have twelve coins. You know that one is fake. The only thing that distinguishes the fake coin from the real coins is that its weight is imperceptibly different. You have a perfectly balanced scale. The scale only tells you which side weighs more than the other side.
   What is the smallest number of times you must use the scale in order to always find the fake coin?
   Use only the twelve coins themselves and no others, no other weights, no cutting coins, no pencil marks on the scale, etc.
   These are modern coins, so the fake coin is not necessarily lighter.
   Presume the worst case scenario, and don’t hope that you will pick the right coin on the first attempt.

(2) I ask Alex to pick any 5 cards out of a deck with no Jokers.
    He can inspect then shuffle the deck before picking any five cards. He picks out 5 cards then hands them to me (Peter can’t see any of this). I look at the cards and I pick 1 card out and give it back to Alex. I then arrange the other four cards in a special way, and give those 4 cards all face down, and in a neat pile, to Peter.
    Peter looks at the 4 cards i gave him, and says out loud which card Alex is holding (suit and number). How?
    The solution uses pure logic, not sleight of hand. All Peter needs to know is the order of the cards and what is on their face, nothing more.

(3) One day three Greek philosophers settled under the shade of an olive tree, opened a bottle of Retsina, and began a lengthy discussion of the Fundamental Ontological Question: Why does anything exist?
    After a while, they began to ramble. Then, one by one, they fell asleep.
    While the men slept, three owls, one above each philosopher, completed their digestive process, dropped a present on each philosopher’s forehead, the flew off with a noisy ”hoot.”
Perhaps the hoot awakened the philosophers. As soon as they looked at each other, all three began, simultaneously, to laugh. Then, one of them abruptly stopped laughing. Why?

(4) I ask people at random if they have two children and also if one is a boy born on a tuesday. After a long search I finally find someone who answers yes. What is the probability that this person has two boys? Assume an equal chance of giving birth to either sex and an equal chance to giving birth on any day.

(5) You have a three gallon and a five gallon measuring device. You wish to measure out four gallons. How do you do it?

(6) After a marathon, judges are comparing notes to determine who won. Use the following to reconstruct the order in which the runners finished the race:
   • Matthew Merryman beat Tom Trent and Jimmy James.
   • Peter Piper beat Jimmy James, Tom Trent and Alan Ardman.
   • Zach Zebra lost to Peter Piper.
   • Graham Goodfellow beat Tom Trent
   • Zach Zebra beat Frank Flintbone.
   • Graham Goodfellow lost to Frank Flintbone and Peter Piper.
   • Tom Trent beat Brian Brick.
   • Alan Ardman beat Zach Zebra, Kevin Kingfisher and Graham Goodfellow.
   • Kevin Kingfisher lost to Graham Goodfellow and Matthew Merryman.
   • Brian Brick beat Kevin Kingfisher.
   • Matthew Merryman lost to Alan Ardman and Zach Zebra.
   • Frank Flintbone beat Tom Trent, Matthew Merryman and Brian Brick.
   • Tom Trent lost to Jimmy James and Alan Ardman.
   • Jimmy James beat Graham Goodfellow and Brian Brick.

(7) A man is 100 meters due south of a bear. He walks 100 yards due east, then faces due north, fires his (water) gun due north, and hits the bear. What color is the bear?

These problems are (adapted) from “What is the name of this book?” Raymond M. Smullyan, Brain Bashers: Logic puzzles, www.folj.com/puzzles, and 2014 class notes of Lawrence Moss