Emory Math Circle
Middle School Session:
Platonic Solids

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Polygons

- 2-D shape
- Border is all straight line segments
  - No Circles!
- Closed: There’s an inside and an outside
Nice Shapes:

Less Nice Shapes:
Regular Polygons

- Convex: Line segments that start and end inside stay inside
  - No Stars!
- All angles are equal
- All sides have the same length
What are all regular polygons?

- There’s exactly one regular polygon with $N$ sides and $N$ angles for each $N \geq 3$. Called an $N$-gon.
  - $N = 3$: equilateral triangle
  - $N = 4$: square
  - $N = 5$: regular pentagon
3D Shapes

- **Vertex** (plural: *vertices*) = corner
- **Edge** = line segment joining two vertices
- **Face** = individual (flat) surface of solid object surrounded by *edges*
- **Polyhedron** (plural: *polyhedra*) = a solid in 3D with flat *faces*, straight *edges*, and sharp *vertices*
For each of your polyhedra, answer the following questions:

- How many vertices, edges, and faces does it have?
- What are the faces? (squares, rectangles, pentagons, etc) Are the faces all the same?
- How many faces meet at each vertex?
Polyhedra
What makes a polyhedron nice?

- All faces are the same regular polygon.
- The same number of faces meet at each vertex.
- Called Platonic Solids.
Are they Platonic Solids?
Properties of Platonic Solids

• Let $M$ be the number of faces that meet at each vertex. Need $M \geq 3$.

• Need all of the sides to be $N$-gons with $N \geq 3$.

• Need sum of all angles at one vertex $< 360^\circ$. 
Sum of interior angles

- Triangle: 180°
- Quadrilateral: 2*180° = 360°
- Pentagon: 3*180° = 540°
- Hexagon: 4*180° = 720°
- N-gon: (N-2)*180°
Size of each angle

- Equilateral triangle: $180^\circ/3 = 60^\circ$
- Square: $360^\circ/4 = 90^\circ$
- Regular Pentagon: $540^\circ/5 = 108^\circ$
- Regular Hexagon: $720^\circ/6 = 120^\circ$
- Regular $N$-gon: $(N - 2) \times 180^\circ/N$
Faces are...

- Triangles:
  - $M = 3$: $3 \times 60^\circ = 180^\circ < 360^\circ$
    - “Tetrahedron”
  - $M = 4$: $4 \times 60^\circ = 240^\circ < 360^\circ$
    - “Octahedron”
  - $M = 5$: $5 \times 60^\circ = 300^\circ < 360^\circ$
    - “Icosahedron”
  - $M = 6$: $6 \times 60^\circ = 360^\circ$ BAD!
Faces are...

- **Squares:**
  - \( M = 3 \): \( 3 \times 90^\circ = 270^\circ < 360^\circ \)
    - “Cube” or “Hexahedron”
  - \( M = 4 \): \( 4 \times 90^\circ = 360^\circ \) BAD!

- **Regular Pentagons:**
  - \( M = 3 \): \( 3 \times 108^\circ = 324^\circ < 360^\circ \)
    - “Dodecahedron”
  - \( M = 4 \): \( 4 \times 108^\circ = 432^\circ \) BAD!

- **Regular Hexagons:**
  - \( M = 3 \): \( 3 \times 120^\circ = 360^\circ \) BAD!