PAlmetto Number Theory Series

Abstracts for September 12 - 13, 2015 Meeting

Olivia Beckwith (Emory University)

Title: The number of parts in certain residue classes of integer partitions

Abstract: We use the Circle Method to derive asymptotic formulas for functions related to the number of parts of partitions in particular residue classes. This is joint work with Michael Mertens.

Saikat Biswas (Arizona State University)

Title: Arithmetic of algebraic tori

Abstract: Suppose that T is an algebraic torus over a global field K, and split over a finite cyclic extension L. We first relate the adele class group of T to the Tamagawa number of T. Next, let X be a principal homogeneous space under T. Then, we relate the arithmetic of X to that of T.

Jeff Breeding-Allison (Fordham University)

Title: Paramodular forms restricted to Humbert surfaces

Abstract: We explain how restricting paramodular forms to Humbert surfaces yields Hilbert modular forms and tensor products of elliptic modular forms for certain principal congruence subgroups. We then explain how to compute spaces of paramodular cusp forms by using Borcherds products as a "valuation" formula and by using restrictions to Humbert surfaces as tractable homomorphisms. This is joint work with Cris Poor and David Yuen.

Shi-Chao Chen (Henan University)

Title: An arithmetic problem on the coefficients of modular forms

Abstract: Let $g(n)$ be a polynomial with integer coefficients and $f(z) = \sum a(n)q^n$ be an integer weight modular form such that $a(n)$ are integers. In this talk, I will give an upper bound the number of n less than x such that $(g(n), a(n)) = 1$.

Victor Guo (University of Missouri)

Title: Almost primes of the form $\lfloor p^c \rfloor$

Abstract: Let $P^c = (\lfloor p^c \rfloor)_{p\in\mathbb{P}}$ with $c > 1,\ c \notin \mathbb{N}$, where P is the set of prime numbers, and $\lfloor\cdot\rfloor$ is the floor function. We show that for every such c there are infinitely many members of P^c having at most $R(c)$ prime factors, giving explicit estimates for $R(c)$ when c is near one and also when c is large.

Wilson Harvey (University of South Carolina)

Title: Covering systems of subsets of the integers

Abstract: The talk will focus on covering systems of certain subsets of the integers, giving historical examples and introducing recent work by the speaker along with his advisors.

Marie Jameson (University of Tennessee)

Title: On p-adic modular forms and the Bloch-Okounkov theorem.

Abstract: TBA

Daniel Kriz (Princeton University)

Title: Congruences of p-adic L-functions

Abstract: I will discuss my result establishing a congruence between the Bertolini-Darmon-Prasanna anticyclotomic p-adic L-function attached to a newform f with reducible residual p-adic Galois representation and the Katz p-adic L-function. From this, there follows a congruence between p-adic Abel-Jacobi images of certain generalized Heegner cycles and products of certain Bernoulli numbers and Euler factors. As an application, one can show that when a semistable elliptic curve $E/\mathbb{Q}$ has reducible mod 3 Galois representation, ranks 0 and 1 each occur with a positive proportion in the quadratic twist family of E, and furthermore one can give explicit families of twists with these ranks. If time permits I will also discuss recent work with Chao Li pertaining to relationships between ranks within quadratic twist families of elliptic curves.

Michael Mertens (Emory University)

Title: Holomorphic projection and mock modular forms

Abstract: We give survey about a specific tool in the theory of harmonic Maaß forms and mock modular forms, holomorphic projection. We describe how to use it to

(1) construct examples of mock modular forms,
(2) prove Eichler-Selberg-type recurrences for Fourier coefficients of mock theta functions,
(3) establish a connection between mock modular forms and a certain kind of L-functions.

Parts of this talk are based on joint work with Ken Ono and Kathrin Bringmann.

Alison Miller (Harvard University)

Title: Algebraic knot invariants, arithmetic invariant theory, and asymptotics

Abstract: Certain knot invariants coming from the Alexander module have natural number-theoretic structure: they can be interpreted as ideal classes in certain rings. In fact, these invariants fit into the structure of arithmetic invariant theory established by Bhargava and Gross.

I will explain this connection, and show how it raises the following asymptotic counting question: how many different possible values can these invariants take for knots whose Alexander polynomial has bounded size? Although this sounds like a topological question, it turns out that it can be made entirely number-theoretic. I will discuss this question for knots of genus 1 and the connection to binary quadratic forms, and mention possibly extensions to higher genus knots.

Jackson Morrow (Emory University)

Title: Composite level images of Galois representations

Abstract: Let E be an elliptic curve defined over $\mathbb{Q}$ without complex multiplication (non-CM) and let ℓ a prime. There is a representation $\rho_{E,\ell}\colon \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \to \mathrm{GL}_2(\mathbb{F}_\ell)$ that describes the Galois action on the ℓ-torsion points of E. Building on recent work of Rouse, Zureick-Brown, and Zywina, we shall talk about composite level modular curves whose rational points classify elliptic curves over $\mathbb{Q}$ with simultaneously non-surjective, composite image of Galois. We shall discuss the techniques used to provably find the rational points on these curves.Finally, we give an application of our results to the study of entanglement fields and present non-CM elliptic curves with peculiar division fields.

Robert Schneider (Emory University)

Title: Partition-theoretic zeta functions

Abstract: We use sums over partitions to compute the values of constants such as π, and to find partition-theoretic analogs of the Riemann zeta function, multiple zeta values, and other objects from classical number theory.

Romyar Sharifi (University of Arizona)

Title: Arithmetic of cyclotomic fields via modular symbols

Abstract: Unit groups and class groups of cyclotomic integer rings were first studied in detail in the mid-19th century in the work of Ernst Kummer on FLT. Even today, many basic questions regarding the structure of these groups remain open. I will describe a connection, in part conjectural, between these groups and modular symbols in the homology of modular curves. I hope to illustrate how this connection can be used to transfer certain statements about the arithmetic of cyclotomic fields to statements about the geometry of modular curves.

Andrew V. Sills (Georgia Southern University)

Title: Parts and subword patterns in integer compositions

Abstract: This is joint work with Brian Hopkins (St. Peter's University), Thotsaporn "Aek" Thanatipanonda (Mahidol University, Thailand) and Hua Wang (Georgia Southern University).

A "composition" of an integer n is a tuple of positive integers that sum to n. Thus the set of all compositions of 4 is

$$\{(4), (31), (13), (22), (211), (121), (112), (1111)\}.$$

Each summand is called a "part" of the composition.

Let $OP(n)$ denote the number of odd parts among all compositions of n. Thus $OP(4) = 14$.

By a "run" in a composition we mean a collection of adjacent equal parts. Thus the composition (22222111311) contains four runs. Let $R(n)$ denote the number of runs among all compositions of n. Notice that $R(4) = 14$.

Our study began with the empirical observations that $OP(n) = R(n)$ and $EP(n+1) = OP(n)$ where $EP(n) =$ the number of even parts among all compositions of n.

From there we were able to prove more general results relating the number of parts in a given residue class modulo m to various subword patterns among all compositions of n.

Kate Thompson (Davidson College)

Title: The sum of four squares over real quadratic number fields.

Abstract: That the sum of four squares represents all positive integers is a well-known and celebrated result–there even is a formula for the number of represented (often presented in undergraduate number theory classes). What happens in the number field analogue? Using Siegel's theory of local densities and Hilbert modular forms, we will answer this question in the case of real quadratic number fields. This includes providing explicit (and, on occasion, sharp) bounds on the Eisenstein coefficients of the associated theta series.

Carl Wang Erickson (Brandeis University)

Title: Ordinary Hecke algebras

Abstract: The Eisenstein component of an ordinary Lambda-adic Hecke algebra carries a module with Galois action, but this module is "not always a representation," i.e. not locally free. However, it does carry a Galois pseudorepresentation, which is the data of characteristic polynomial coefficients. Joint work with Preston Wake has produced an universal ordinary pseudodeformation ring, which can be compared with the Hecke algebra. I will discuss how this comparison can be used to establish new cases of Sharifi's conjecture and new proofs of the residually reducible modularity results of Skinner and Wiles, conditional on expected conjectures on class groups.

John Webb (James Madison University)

Title: On calculating bases of modular forms

Abstract: We will report on the results of a summer research project to create a new algorithm for calculating bases of integer weight modular forms on $\Gamma_0(N)$ for certain values of N. While the algorithms currently in use by Magma and Sage rely on modular symbols, our bases are created using eta quotients. We will report on performance results and on progress towards a hybrid algorithm which utilizes the best attributes of each method.

Robert Wilcox (University of South Carolina)

Title: An explicit universal Hilbert set of asymptotic density 1.

Abstract: A universal Hilbert set is an infinite set $\mathbb{H} \subset \mathbb{Z}$ with the following property: for any $f(x,y) \in \mathbb{Z}[x,y]$ irreducible in $\mathbb{Q}[x,y]$, the polynomial $f(x,h) \in \mathbb{Z}[x]$ is irreducible in $\mathbb{Q}[x]$ for all but finitely many $h \in \mathbb{H}$. Bilu in 1996 and Debes and Zannier in 1998 showed non-constructively the existence of such a set of asymptotic density 1. In this talk, we describe a correspondence between irreducible bivariate polynomials and integer points on a curve. Using Siegel's theorem on integer points, we present an explicit construction of a universal Hilbert set of asymptotic density 1.

Zhengyao Wu (Emory University)

Title: Springer's theorem over p-adic curves

Abstract: Let F be the function field of one variable over a p-adic curve, M/F an odd degree extension, D a division F-algebra with an involution, h a hermitian or skew-hermitian space over D. If h_M has a nontrivial zero, then h has a nontrivial zero over F.

Shouwu Zhang (Princeton University)

Title: Congruent number problem and L-functions

Abstract: A thousand years old problem is to determine which positive integers are congruent numbers, i.e. those numbers which could be the areas of right angled triangles with sides of rational lengths. This problem has some beautiful connections with elliptic curves and L-functions. In fact by the Birch and Swinnerton-Dyer conjecture, all $n = 5, 6, 7$ mod 8 should be congruent numbers, and most of $n = 1, 2, 3$ mod 8 should not be congruent numbers. In this lecture, I will explain these connections and some recent developments.

Scott Zinzer (West Virginia Wesleyan College)

Title: On the Iwasawa μ-invariant of a two-variable p-adic L-function.

Abstract: In this talk, I describe some results contained in my Ph.D. thesis on the Iwasawa invariants of certain multivariate p-adic measures. As an application, I describe a measure-theoretic verification of the vanishing of the Iwasawa μ-invariant of Yager's two-variable p-adic L-function.