Combinatorics Seminar

On-line Ramsey Theory

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Let c,s,t be positive integers. We call an s-uniform a hypergraph an s-graph. The (c,s,t)\emph{-Ramsey game} is played by Builder and Painter. Play begins with an s-graph $G_{0}=(V,E_{0})$, where $E_{0}=\emptyset$ and V is determined by Builder. On the ith round Builder constructs a new edge e_{i} and sets $G_{i}=(V,E_{i})$, where $E_{i}% =E_{i-1}\cup\{e_{i}\}$. Painter responds by coloring e_{i} with one of c colors. Builder wins if Painter eventually creates a monochromatic copy of K_{s}^{t}, the complete s-graph on t vertices; otherwise Painter wins when she has colored all possible edges. We extend the definition of coloring number to s-graphs in a natural way so that $\chi(G)\leq\operatorname*{col}(G)$ for any s-graph G and then show that Builder can win (c,s,t)-Ramsey game while building a hypergraph with coloring number at most $\operatorname*{col}(K_{s}^{t})$. This relates to, but does not answer, a question of Kurek, Ruci\'{n}ski and R\"{o}dl as to whether Builder can win the $(2,2,t)$-Ramsey game while constructing an s-graph with substantially fewer than $\binom{\operatorname*{Ram}(t)}{2}$ edges. Our main tool is the analysis of the following new game. Let p be positive integers with $s\leq p$. The (p,s,t)\emph{-survival game} is played by two players, \emph{Presenter} and \emph{Chooser}. Play begins with the s-graph $H_{0}=(S_{0},E_{0})$ as in the Ramsey game. At the beginning of the ith round the players will have constructed an s-graph $H_{i-1}=(S_{i-1},E_{i-1})$. During the ith round they construct $H_{i}=(S_{i},E_{i})$ as follows. Presenter plays by presenting a p-subset $P_{i}\subseteq S_{i-1}$. Chooser responds by choosing an s-set $X_{i}\subseteq P_{i}$. The remaining vertices in $P_{i}-X_{i}$ are discarded, leaving $S_{i}=S_{i-1}-(P_{i}-X_{i})$ and \[E_{i}=(E_{i-1}\cup\{X_{i}\})-\left\{ X_{j}\in E_{i-1}:X_{j}\nsubseteq S_{i}\right\} . \] Presenter wins if H_{i} contains the complete s-uniform hypergraph for some i; otherwise Chooser wins when $\left\vert S_{i}\right\vert <t$. \begin{theorem} \label{S}For all positive integers p,s,t with $s\leq p$, Presenter has a winning strategy in the (p,s,t)-survival game. \end{theorem} This is joint work with Goran Konjevod.

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