Numerical Analysis and Scientific Computing Seminar

Physics-based preconditioners for solving PDEs in highly heterogeneous media

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Abstract: Eigenvalues of smallest magnitude become a major bottleneck for iterative solvers especially when the underlying physical properties have severe contrasts. These contrasts are commonly found in many applications such as composite materials, geological rock properties and thermal and electrical conductivity.

The main objective of this work is to construct a method as algebraic as possible that could efficiently exploit the connectivity of highly heterogeneous media in the solution of diffusion operators. We propose an algebraic way of separating binary-like systems according to a given threshold into high- and low-conductivity regimes of coefficient size $O(m)$ and $O(1)$, respectively where $m \gg 1$. The condition number of the linear system depends both on the mesh size Δx and the coefficient size m. For our purposes, we address only the m dependence since the condition number of the linear system is mainly governed by the high-conductivity subblock. Thus, the proposed strategy is inspired by capturing the relevant physics governing the problem. Based on the algebraic construction, a two-stage preconditioning strategy is developed as follows: (1) a first stage that comprises approximation to the components of the solution associated to small eigenvalues and, (2) a second stage that deals with the remaining solution components with a deflation strategy (if ever needed). The deflation strategies are based on computing near invariant subspaces corresponding to smallest and deflating them by the use of recycled the Krylov subspaces.

Due to its algebraic nature, the proposed approach can support a wide range of realistic geometries (e.g., layered and channelized media). Numerical examples show that the proposed class of physics-based preconditioners are more effective and robust compared to a class of Krylov-based deflation methods on highly heterogeneous media. We also report singular perturbation analysis of the stiffness matrix and the impact of the number of high-conductive regions on various matrices.

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