The bandwidth conjecture of Bollobás and Komlós

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Abstract: The study of sufficient degree conditions, on a given graph $G$, which imply that $G$ contains a particular spanning subgraph $H$ is one of the central areas in graph theory. A well known example of such a result is Dirac’s theorem. It asserts that any graph $G$ on $n$ vertices with minimum degree at least $n/2$ contains a spanning, so called Hamiltonian, cycle.

In my talk we discuss related results for graphs $H$ of bounded maximum degree and small bandwidth. In particular we show that: For all integers $k$ and $\Delta$ and $\gamma > 0$ there exist a constant $\beta > 0$ such that for sufficiently large $n$ the following holds. If $G$ is an $n$-vertex graph with minimum degree $\delta(G) \geq ((k - 1)/k + \gamma)n$, then it contains a spanning copy of every $k$-chromatic $n$-vertex graph $H$ with maximum degree $\Delta(H) \leq \Delta$ and bandwidth $bw(H)$ at most $\beta n$, where $bw(H) = \min_{\sigma} \max_{uv \in E(H)} |\sigma(u) - \sigma(v)|$ with the minimum ranging over all bijections from $V(H)$ to $[n]$. This settles a conjecture of Bollobás and Komlós. It is known that the minimum degree condition on $G$ is asymptotically best possible.

The proof is based on Szemerédi’s regularity lemma and the so called Blow-up lemma. This is joint work with Julia Böttcher and Anusch Taraz from TU-München.

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