Dissertation Defense

Pólya's Theorem with Zeros

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Abstract

Let $\mathbb{R}[X] = \mathbb{R}[X_1, \ldots, X_n]$ and let Δ_n denote the standard n-simplex $\{(x_1, \ldots, x_n) \in \mathbb{R}^n \mid x_i \geq 0, \sum_i x_i = 1\}$. Pólya's Theorem says that if a form (homogeneous polynomial) $p \in \mathbb{R}[X]$ is positive on Δ_n, then for sufficiently large $N \in \mathbb{N}$ all of the coefficients of $(X_1 + \ldots + X_n)^N p$ are positive. In 2001, Powers and Reznick established an explicit bound for the N in Pólya's Theorem. The bound depends only on information about p, namely the degree and the size of the coefficients of p, and the minimum value of p on the simplex.

This work in this talk is part of an ongoing project, started by Powers and Reznick in 2006, to understand exactly when Pólya's Theorem holds if the condition "positive on Δ_n" is relaxed to "nonnegative on Δ_n", and to give bounds in this case. We show that if a form p satisfies a relaxed version of Pólya's Theorem, then the set of zeros of p is a union of faces of the simplex. We characterize forms which satisfy a relaxed version of Pólya's Theorem and have zeros on vertices. Finally, we give a sufficient condition for forms with zero set a union of two-dimensional faces of the simplex to satisfy a relaxed version of Pólya's Theorem, with a bound.

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