Algebra Seminar

From nilpotent matrices to support varieties for Lie algebras

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Abstract: It’s easy to describe the set of $n$-by-$n$ complex matrices whose $r$-th power is zero: each is conjugate to a Jordan canonical form matrix with all eigenvalues 0 and blocks of size at most $r$. In this talk I will discuss work done by the University of Georgia VIGRE Algebra Group on generalizations of this problem. We will consider an arbitrary simple algebraic group $G$ over an algebraically closed field, along with certain embeddings of its Lie algebra $g$ into $n$-by-$n$ matrices, and describe the set of elements of $g$ whose $r$-th power is zero, in terms of $G$-conjugacy classes. As a corollary, when the characteristic is $p$, we obtain a description of the “restricted nullcone” of $g$ (the case $r = p$). When $p$ is a “good prime” (not too small), this verifies, by much more elementary methods, a 2003 result of Carlson, Lin, Nakano, and Parshall. And when $p$ is “bad” (very small), our results are new.

Support varieties are geometric objects associated to cohomology rings and representations. In the case of modular representations of Lie algebras, the supports are subvarieties of the restricted nullcone. In the last part of my talk, I’ll briefly define support varieties for Lie algebras, and then discuss how our VIGRE Algebra Group identified explicitly the support varieties of induced modules in the bad prime setting. This extends the 2002 verification by Nakano, Parshall, and Vella of a 1987 conjecture of Jantzen for good primes.

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