Abstract: There has been a long history on how to construct all rational solutions of cubic equations of two variables like $y^2 = 4x^3 + ax + b$. We will see how imaginary quadratic fields, together with the study of some $L$-functions, can be used to partially solve the question. We will also mention some recent progress that leads us to a broader perspective.