Abstract: I will discuss the following theorem: for any fixed complex numbers $a$ and $b$, the set of complex numbers $c$ for which both $a$ and $b$ both have finite orbit under iteration of the map $z \rightarrow z^2 + c$ is infinite if and only if $a^2 = b^2$. I will explain the motivation for this result and give an outline of the proof. The main arithmetic ingredient in the proof is an adelic equidistribution theorem for preperiodic points over product formula fields, with non-archimedean Berkovich spaces playing an essential role. This is joint work with Laura DeMarco, relying on earlier joint work with Robert Rumely.