Abstract: In this talk we consider an extremal problem regarding multigraphs with edge multiplicity bounded by a positive integer $q$. The number $a$, $0 \leq a < q$ is a jump for $q$ if, for any positive $e$, any integer $m$, and any $q$-multigraph on $n > n_0(e, a)$ vertices and at least $(a + e)(n(n - 1)/2)$ edges, counting multiplicity, there is a subgraph on $m$ vertices and at least $(a + c)(m(m - 1)/2)$ edges, where $c = c(a)$ does not depend on $e$ or $m$. The Erdős-Stone theorem implies that for $q = 1$ every $a \in [0, 1)$ is a jump. The problem of determining the set of jumps for $q \geq 2$ appears to be much harder. In a sequence of papers by Erdős, Brown, Simonovits and separately Sidorenko, the authors established that every $a$ is a jump for $q = 2$ leaving the question whether the same is true for $q \geq 3$ unresolved. A later result of Rödl and Sidorenko gave a negative answer, establishing that for $q \geq 4$ some values of $a$ are not jumps. The problem of whether or not every $a \in [0, 3)$ is a jump for $q = 3$ has remained open. We give a partial positive result in this talk, proving that every $a \in [0, 2)$ is a jump for all $q \geq 3$. Additionally, we extend the results of Rödl and Sidorenko by showing that, given any rational number $r$ with $0 < r \leq 1$, that $(q - r)$ is not a jump for any $q$ sufficiently large. This is joint work with Paul Horn and Vojtěch Rödl.

Friday, November 18, 2011, 4:00 pm
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