Algebra Seminar

The distribution of 2-Selmer ranks and additive functions

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Abstract: The problem of determining the distribution of the 2-Selmer ranks of quadratic twists of an elliptic curve has received a great deal of recent attention, both in works conjecturing distributions and in those providing solutions; in both cases, the nature of the two-torsion of the elliptic curve plays a crucial role. In particular, if $E/\mathbb{Q}$ has full two-torsion, the distribution is known, due to work of Heath-Brown, Swinnerton-Dyer, and Kane, and if $E$ possesses no two-torsion, then, again, the distribution is known, due to work of Klagsbrun, Mazur, and Rubin, though with the caveat that one arranges discriminants in a non-standard way. In stark contrast to these two cases, we show that if $K$ is a number field and $E/K$ is an elliptic curve with partial two-torsion, then no limiting distribution on 2-Selmer ranks exists. We do so by showing that, for any fixed integer $r$, at least half of the twists of $E$ have 2-Selmer rank greater than $r$, and we establish an analogous result for simultaneous twists, either for multiple elliptic curves twisted by the same discriminant or for a single elliptic curve twisted by a tuple of discriminants. These results depend upon connecting the 2-Selmer rank of twists to the values of an additive function and then establishing results analogous to the classical Erdős-Kac theorem. This work is joint with Zev Klagsbrun.

Thursday, November 21, 2013, 5:00 pm
Mathematics and Science Center: W306

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