Combinatorics Seminar

On Erdos’ conjecture on the number of edges in 5-cycles

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Abstract: Erdos, Faudree, and Rousseau (1992) showed that a graph on \( n \) vertices and at least \( \frac{n^2}{4} + 1 \) edges has at least \( 2\left\lfloor \frac{n}{2} \right\rfloor + 1 \) edges on triangles and this result is sharp. They also considered a conjecture of Erdos that such a graph can have at most \( \frac{n^2}{36} \) non-pentagonal edges with an extremal graph having two components, a complete graph on \( \left\lfloor \frac{2n}{3} \right\rfloor + 1 \) vertices and a complete bipartite graph on the rest. This was mentioned in other papers of Erdos and also it is No. 27 in Fan Chung’s problem book.

In this talk we give a graph of \( \frac{n^2}{4} + 1 \) edges with much more, namely \( \frac{n^2}{8}(2 + \sqrt{2}) + O(n) = \frac{n^2}{27.31} \) pentagonal edges, disproving the original conjecture. We show that this coefficient is asymptotically the best possible.

Given graphs \( H \) and \( F \) let \( E_0(H,F) \) denote the set of edges of \( H \) which do not appear in a subgraph isomorphic to \( F \), and let \( h(n,e,F) \) denote the maximum of \( |E_0(H,F)| \) among all graphs \( H \) of \( n \) vertices and \( e \) edges. We asymptotically determine \( h(n, cn^2, C_3) \) and \( h(n, cn^2, C_5) \) for fixed \( c, \frac{1}{4} < c < \frac{1}{2} \). For \( 2k + 1 \geq 7 \) we establish the conjecture of Erdos et al. that \( h(n, cn^2, C_{2k+1}) \) is obtained from the above two-component example.

One of our main tools (beside Szemeredi’s regularity) is a new version of Zykov’s symmetrization what we can apply for more graphs, simultaneously.

Friday, December 6, 2013, 4:00 pm
Mathematics and Science Center: W306

This is joint work with Zeinab Maleki.

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