Continuous analogues of methods used to calculate component groups of Jacobians

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Abstract: Let K be complete, discretely-valued field and let X be a smooth projective K-curve equipped with a semistable model over the valuation ring. A series of classical theorems, mostly due to Raynaud, give two ways of calculating the component group of the Jacobian J of X: one using the intersection matrix on the special fiber of the model of X, and the other using cycles on its incidence graph G. These calculations can be interpreted in terms of divisors on G (in the sense of Baker-Norine) and the uniformization theory of G, respectively. If K is complete and non-Archimedean but not discretely valued, these theorems are no longer applicable, as Nron models do not exist in this situation. Replacing the component group with the skeleton of J (in the sense of Berkovich), a principally polarized real torus canonically associated to J, and the incidence graph with a skeleton Gamma of X, a metric graph, we will prove "continuous" analogues of these theorems. Specifically, we will show that the Jacobian of Gamma is canonically identified with the skeleton of J as principally polarized real tori, in a way that is compatible with the descriptions of the two Jacobians in terms of divisors and in terms of uniformizations. As a consequence, we will show that, when K is algebraically closed, essentially any piecewise-linear function on Gamma is the restriction to Gamma of $-\log |f|$, where f is a nonzero rational function on X.

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