Abstract: In this thesis, the author proves theorems relating to three different areas in the study of elliptic curves: torsion subgroups over number fields, Selmer groups of elliptic curves, and composite level images of Galois. In particular, the thesis contains theorems completing the classification of possible torsion subgroups for elliptic curves defined over cubic number fields; bounding the order of $\ell$-Selmer groups for twists of elliptic curves defined over number fields of small degree; and determining the possibilities, indicies, and occurrences of composite level images of Galois for elliptic curves defined over $\mathbb{Q}$.