Abstract: We present the algebro-geometric theory underlying the classical transformation laws of theta functions with respect to the action of symplectic matrices on Sigel’s upper half-space. More precisely, we explain how the theta multiplier, the half-integral weight automorphy factor and the Weil representation occurring in the classical transformation laws all have a geometric origin, that is, they can all be constructed within a given moduli problem on abelian schemes. To do so, we introduce and study new algebro-geometric constructions such as theta multiplier bundles, metaplectic stacks and bundles of half-forms, which could be of independent interest. Applications include a geometric theory of modular forms of half-integral (in the sense of Shimura), and their generalizations to higher degree.