Dissertation Defense

On Saturation Spectrum

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Abstract: Given a graph \( H \), we say a graph \( G \) is \( H \)-saturated if \( G \) does not contain \( H \) as a subgraph and the addition of any edge not already in \( G \) results in \( H \) as a subgraph. The question of the minimum number of edges of an \( H \) saturated graph on \( n \) vertices, known as the saturation number, and the question of the maximum number of edges possible of an \( H \)-saturated graph, known as the Turn number, has been addressed for many different types of graphs. We are interested in the existence of \( H \)-saturated graphs for each edge count between the saturation number and the Turn number.

We determine the saturation spectrum of \((K_t-e)\)-saturated graphs and \( F_t \)-saturated graphs. Let \((K_t-e)\) be the complete graph minus one edge. We prove that \((K_t-e)\)-saturated graphs do not exist for small edge counts and construct \((K_t-e)\)-saturated graphs with edge counts in a continuous interval. We then extend the constructed \((K_t-e)\)-saturated graphs to create \((K_t-e)\)-saturated graphs. Let \( F_t \) be the graph consisting of \( t \) edge-disjoint triangles that intersect at a single vertex \( v \). We prove that \( F_2 \)-saturated graphs do not exist for small edge counts and construct a collection of \( F_2 \)-saturated graphs with edge counts in a continuous interval. We also establish more general constructions that yield a collection of \( F_t \)-saturated graphs with edge counts in a continuous interval.

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