Abstract:

Herzog, Vladoiu, and Zheng posed the following question: Given a positive integer $n$, does there exist a partition $\mathcal{P}_n$ of the non-empty subsets of \{1, 2, \ldots, n\} into intervals, so that $|B| \geq n/2$ for each interval $[A, B]$ in $\mathcal{P}_n$. Their motivation involves a 1982 conjecture of Richard Stanley concerning what is now called the Stanley depth of a module. We answer this question in the affirmative by first embedding it in a stronger result. We then provide two alternative proofs of this second result. The two proofs use entirely different methods and yield non-isomorphic partitions. This presentation will focus on the elegant combinatorial arguments for constructing the partitions and will be accessible to undergraduates.