Rotationally Symmetric Planes in Comparison Geometry

Global Riemannian geometry is concerned with relating geometric data such as curvature, volume, and radius to topological data. Cheeger-Gromoll showed that any noncompact complete manifold M with nonnegative sectional curvature contains a boundaryless, totally convex, compact submanifold S, called a soul, such that M is homeomorphic to the normal bundle over S. In the first part of our talk, we show that if M is a rotationally symmetric plane M_m, defined by metric $dr^2 + m^2(r)d\theta^2$, then the set of souls is a closed geometric ball centered at the origin, and if furthermore M_m is a von Mangoldt plane, then the radius of this ball can be explicitly determined. We show that the set of critical points of infinity in M_m is equal to this set of souls, and we make some additional observations on the set of critical points of infinity when M_m is von Mangoldt with a point at which the sectional curvature is negative. We close out the first part of the talk with showing that the slope $m'(r)$ of M_m near infinity can be prescribed with any number in $(0, 1]$. In the second part of our talk, we extend results in a paper by Kondo-Tanaka in which the authors generalize the Toponogov Comparison Theorem such that an arbitrary noncompact manifold M is compared with a rotationally symmetric plane M_m satisfying certain conditions to establish that M is topologically finite. We substitute one of the conditions for M_m with a weaker condition and show that our method using this weaker condition enables us to draw further conclusions on the topology of M. We also completely remove one of the conditions required for the Sector Theorem, one of the principal results in the same paper.