Combinatorics Seminar

What is Ramsey-equivalent to a clique?

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Abstract: A graph G is *Ramsey* for H if every two-colouring of the edges of G contains a monochromatic copy of H. Two graphs H and H' are Ramsey-equivalent if every graph G is Ramsey for H if and only if it is Ramsey for H'. In this paper, we study the problem of determining which graphs are Ramsey-equivalent to the complete graph, K_k. A famous theorem of Nesetril and Rodl implies that any graph H which is Ramsey-equivalent to K_k must contain K_k. We prove that the only connected graph which is Ramsey-equivalent to K_k is itself. This gives a negative answer to the question of Szabo, Zumstein, and Zurcher on whether K_k is Ramsey-equivalent to $K_k \cdot K_2$, the graph on $k+1$ vertices consisting of K_k with a pendent edge.

In fact, we prove a stronger result. A graph G is Ramsey minimal for a graph H if it is Ramsey for H but no proper subgraph of G is Ramsey for H. Let $s(H)$ be the smallest minimum degree over all Ramsey minimal graphs for H. The study of $s(H)$ was introduced by Burr, Erdos, and Lovasz, where they show that $s(K_k) = (k-1)^2$. We prove that $s(K_k \cdot K_2) = k-1$, and hence K_k and $K_k \cdot K_2$ are not Ramsey-equivalent.

We also find the values of $s(H)$ for various other graphs H, such as 3-connected bipartite graphs. Joint work with Jacob Fox, Anita Liebenau, Yury Person, and Tibor Szabo.

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