Math 111 Diagnostic Test

Instructions: Take this test without consulting your book and without spending too much time on any one question. The goal is for me to get an idea of where you stand, not for you to impress me by getting a perfect score. If you wish to consult the book before taking the test to refresh your memory, that is perfectly reasonable and even recommended. Just make sure to put everything away before you look at this test. Furthermore, my expectation is not necessarily that this is all comfortable for you, I just want to see where your limits are. We will spend some time reviewing all of this material in class.

For reference, here are some of the topics: factoring and simplifying algebraic expressions, fractions, exponents, solving equations (and the quadratic formula), equation of a line (point-slope form), and functions (domain, range).

You don’t have to worry about syntax, but be very clear in your solutions. Use notation such as ^ for exponentiation, / for fractions, | for absolute value, and pi for \( \pi \). Use parentheses whenever there might be confusion, such as \( x^{(1/2)} \) for \( x^{1/2} \) or \( (3x - 7)/(2x + 6) \) for \( \frac{3x - 7}{2x + 6} \). There is no need to use a symbol to denote multiplication, but if you feel like it, use \( * \) instead of \( x \). For square roots you may use an exponent or write \( \sqrt{x} \), but for cube roots simply use exponents.

Note that there might be some things that I ask you to simplify that are already in their simplest form.

To submit your answers, go to the class webpage and click on “Diagnostic Test.” Or just click here:

https://docs.google.com/spreadsheet/viewform?formkey=dE1RZWx5UDM1bkZDwXVmenV0UG9yb2c6MQ

1. Simplify the following expressions:
   (a) \( \frac{a^{191}}{a^{91}} \) (b) \( a^2 a^{-5} \) (c) \( a^3 b^4 \) (d) \( \left( \frac{6x^2 y}{y^{3/2} \sqrt{x}} \right)^{-2} \) (e) \( \sqrt[3]{x^3 y^5} \).

2. Expand and simplify the following expressions:
   (a) \((3x + 5)^2 \) (b) \((x + 3)(2x^2 + 5x + 1) \).

3. Factor the following expressions:
   (a) \( x^2 - 36 \) (b) \( 2x^2 + 11x - 6 \) (c) \( 2x^{3/2} - 8x^{1/2} + 6x^{-1/2} \).

Remark: In part (c), I expect you to factor something out so that you are multiplying it by a polynomial. Then you should be able to further factor the polynomial.

4. Simplify the following rational functions:
   (a) \( \frac{x^3}{x^3 + 1} \) (b) \( \frac{1}{x} - \frac{1}{y} \) (c) \( \frac{3x}{x^2 - 1} + \frac{1}{x + 1} \).
5. Rationalize the denominator of \( \frac{6}{3 - \sqrt{3}} \).

6. Solve for \( x \) in the following equations:

   \( (a) \ x^2 + 6x = -9 \quad (b) \ 2x^2 - 3x - 1 = 0 \quad (c) \ \frac{-5x}{x - 1} = 9x. \)

   **Remark:** Be careful in part (c). There are two solutions!

7. For what range of \( x \) are the following inequalities true?

   \( (a) \ |x - 2| < 5 \quad (b) \ (x - 1)(x - 2) > 0. \)

8. What is the equation of a circle centered at the point \((-2, 1)\) with radius 5?

9. Give the equation of a line passing through \((1, 1)\) with slope \(-2\) in point-slope form.

10. What function is this the graph of? Give your answer in the form of \( f(x) = \) [insert function here].

![Graph of a function](image)

11. Give the domains of the following functions. If you would like to say something like “all\( x \) except for \( x = 3 \)” you may certainly do that, but it’s easier to just write \( x \neq 3 \).

   \( (a) \ \frac{x - 1}{x^2 + 2x - 3} \quad (b) \ \sqrt{-2x + 6}. \)
12. Give the range of the following functions:

(a) \( \sqrt{x} \), where \( x \geq 0 \)  
(b) \( x^2 + 6x + 8 \).

13. Let \( f(x) = x^2 + 2 \) and let \( g(x) = \sqrt{x} \). What are (a) \( f \circ g(x) \) and (b) \( g \circ f(x) \)?

14. For what values of \( \theta \) between 0 and \( 2\pi \) is \( \cos(\theta) = 0 \)?