1. $g'(5) = 0$, but $g'$ is always positive.

2. (b) One (c) No, because $f'$ is decreasing for all $x$ ($f'' < 0$)

3. Let $f(x) = 2\sqrt{x} - 3 + 1/x$. Then for $x > 1$, $f'(x) > 0$ (because $1/\sqrt{x} > 1/x^2$ for $x > 1$), and $f(1) = 0$, so for $x > 1$, $f$ is increasing from 0.

4. There is a local max at $x = 0$, a local min at $x = 4/7$ and neither at the critical point $x = 1$.

5. 

6. 

7. He should charge $900 per unit. Solution: $R = (800 + 10x)(100 - x)$ is the revenue the landlord would make if he raises the rent by $10. We can expand this to $R = 80000 + 200x - 10x^2$. Differentiating with respect to $x$ we get $R' = 200 - 20x$, which equals 0 when $x = 10$. This corresponds to $R = 81000$. We need to verify that this is a maximum of $R$. Since $R$ is concave down everywhere ($R'' = -20$), we do indeed get an absolute maximum at $x = 10$. Alternatively you can plug in the “endpoints” which are $x = 0$ and $x = 100$, since $0 \leq x \leq 100$. This gives a revenue of 80000 and 0 respectively, so 81000 is the absolute maximum. Then the best rent to charge is 900.

8. $C = 54$ is the minimum cost. Solution: Let $w$ be the width of the base, $2w$ be the length of the base (as given by the problem), and $h$ be the height of the box. Then $C = 1(2w^2) + 2(2wh + 4wh)$ gives the cost of constructing the box, since the area of the base is $2w^2$ and costs $1 per square meter and the two pairs of sides, which have area $wh$ and $2wh$, cost $2$ per square meter. We can simplify this to $C = 2w^2 + 12wh$. Since we know that $V = 2w^2h = 18$, we know that $h = 9/w^2$. Plugging this into $C$ we get $C = 2w^2 + 108/w$. Then $C' = 4w - 108/w^2$, which equals 0 when $w = 3$. We need to justify that 3 is a minimum, but this is true by the first derivative test. Plugging $w = 3$ back into $C$ we get $C = 2(9) + 108/3 = 54$.

9. $\frac{db}{dt} = -8/5$ Solution: Let $b$ be the length of the base, let $h$ be the altitude, and let $A = bh/2$ be the area of the triangle. Then if we differentiate the formula for the area with respect to $t$, by the product rule we get

$$\frac{dA}{dt} = \frac{1}{2} \left[ b \frac{dh}{dt} + h \frac{db}{dt} \right].$$

We know that $\frac{dA}{dt} = 2$ and $\frac{dh}{dt} = 1$, and we want to find $\frac{db}{dt}$ when $h = 10$ and $A = 100$. This means we need to find out what $b$ is when $h = 10$ and $A = 100$ so we can plug it into our equation. Since $A = bh/2$, when $h = 10$ and $A = 100$, $b = 20$. So we have

$$2 = \frac{1}{2} \left[ (20)(1) + 10 \frac{db}{dt} \right].$$

Solving for $\frac{db}{dt}$ we get $\frac{db}{dt} = -4/5$. 

1
10. \( \frac{d\theta}{dt} = -1/8 \) Solution: Let \( x \) be the distance of the bottom of the ladder from the wall and let \( \theta \) be the angle of elevation. Then \( \cos \theta = x/10 \) since the ladder is 10 feet long. Differentiating with respect to \( t \) we get
\[
-\sin \theta \frac{d\theta}{dt} = \frac{1}{10} \frac{dx}{dt}.
\]
We are given that \( \frac{dx}{dt} = 1 \) and we want to find \( \frac{d\theta}{dt} \) when \( x = 6 \). When \( x = 6 \), the top of the ladder is 8 feet above the ground (by the Pythagorean theorem), so \( \sin \theta = 8/10 = 4/5 \) when \( x = 6 \). So we can plug this back into our equation and we get that \( \frac{d\theta}{dt} = -1/8 \).

11. (a) -3 (b) -\infty (c) -1 (d) -\infty

12. \( s(t) = -\cos t + \frac{1}{3}t^3 - \frac{x^3}{3} \)

13. \( f(x) = x^5 + x^4 + 2x^2 - 7x + 8 \)

14. (a) \( -x \sin(-x) = x \sin x \) (but the first answer is perfectly acceptable) (b) \( 2x \sin(x^6) - \sin(x^3) \)

15. (a) 0 (b) 17/6 (c) 2