Math 111 - Quiz 1

September 7, 2012

Instructions: Show all of your work and mark your answers clearly. Each problem is worth 5 points.

1. Find the domain of the function \( f(x) = \frac{x + 4}{x^2 - 9} \).

\[ x^2 - 9 = 0 \implies (x - 3)(x + 3) = 0 \]
\[ \implies x = \pm 3 \]

So the domain of \( f \) is \( \{ x \neq \pm 3 \} \),
or \( x \) in \( (-\infty, -3) \cup (-3, 3) \cup (3, \infty) \),
or \( \exists x \in \mathbb{R} \mid x \neq \pm 3 \).

2. Find the domain of the function \( F(p) = \sqrt{2 - \sqrt{p}} \).

First, we need \( p \geq 0 \), so that \( \sqrt{p} \) is defined.

Then \( 2 - \sqrt{p} \geq 0 \implies \sqrt{p} \leq 2 \implies p \leq 4. \)

So the domain is \( \{ 0 \leq p \leq 4 \} \).
3. Let \( f(x) = 1 - 3x \) and \( g(x) = \cos x \). Find the functions \( f \circ g, g \circ f, f \circ f, \) and \( g \circ g \).

Be sure to label your answers!

\[
\begin{align*}
(f \circ g)(x) &= f(\cos x) = 1 - 3 \cos x. \\
(g \circ f)(x) &= g(1 - 3x) = \cos(1 - 3x). \\
(f \circ f)(x) &= f(1 - 3x) = 1 - 3(1 - 3x) = 8x - 2. \\
(g \circ g)(x) &= g(\cos x) = \cos(\cos x).
\end{align*}
\]

4. Let \( F(x) = \frac{\sqrt{x}}{1 + \sqrt{x}} \). Express \( F \) as \( f \circ g \).

Let \( f(x) = \frac{x}{1 + x} \) and \( g(x) = \frac{3}{\sqrt{x}} \).

Then \( F = f \circ g \).