1. Differentiate the function $y = \sin(x \cos x)$ with respect to $x$.

\[
\frac{dy}{dx} = \frac{d}{dx}(x \cos x) \cdot \cos(x \cos x)
\]

\[
= (x (-\sin x) + \cos x) \cos(x \cos x).
\]

2. Find $dy/dx$ by implicit differentiation if $x^4(x + y) = y^2(3x - y)$.

\[
\frac{d}{dx} \left( x^4(x + y) \right) = \frac{d}{dx} \left( y^2(3x - y) \right)
\]

\[
\Rightarrow \quad x^4(1 + y') + 4x^3(x + y) = y^2(3 - y') + 2yy' (3x - y)
\]

\[
\Rightarrow \quad x^4y' + y^2y' - 2yy'(3x - y) = 3y^2 - x^4 - yx^3(x + y)
\]

\[
\Rightarrow \quad y' = \frac{3y^2 - x^4 - yx^3(x + y)}{x^4 + y^2 - 2y(3x - y)}
\]
3. Find $y''$ by implicit differentiation if $x^3 + y^3 = 1$.

$$3x^2 + 3y^2 y' = 0 \implies y' = \frac{-3x^2}{3y^2} = \frac{-x^2}{y^2} = -x^2 y^{-2}$$

So

$$y'' = -x^2 (-2y^{-3} y') + (-2x) y^{-2} = -x^2 (-2y^{-2} (\frac{-x^2 y^{-2}}{y'})) - 2xy^{-2}.$$ 

4. A cylindrical tank with radius 5 m is being filled with water at a rate of 3 m$^3$/min. How fast is the height of the water increasing? (The volume of a cylinder is given by $V = \pi r^2 h$).

$$\frac{dV}{dt} = 3 \ m^3/\text{min}$$

$$V = \pi r^2 h = 25 \pi h$$

$$\implies \frac{dV}{dt} = 25 \pi \frac{dh}{dt}$$

So

$$\frac{dh}{dt} = \frac{1}{25 \pi} \frac{dV}{dt} = \frac{3}{25 \pi} \ m/\text{min}$$