Math 111 - Quiz 6

October 26, 2012

Instructions: Show all of your work and mark your answers clearly.

1. (5 points) Let \( f(x) = 2x^3 - 3x^2 - 12x + 1 \). Find the absolute maximum and absolute minimum values of \( f \) on the interval \([-2, 3]\).

\[
\begin{align*}
    f'(x) &= 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x - 2)(x + 1) = 0 \\
    \text{Critical points: } & x = 2, \ x = -1 \\
    f(2) &= 2(8) - 3(4) - 12(2) + 1 = 16 - 12 - 24 + 1 = -19 \quad \text{Absolute min} \\
    f(-1) &= 2(-1) - 3(-1) - 12(-1) - 1 = -2 + 3 + 12 + 1 = 8 \quad \text{Absolute max} \\
    f(-2) &= 2(-8) - 3(-4) - 12(-2) + 1 = -16 - 12 + 24 + 1 = -3 \\
    f(3) &= 2(27) - 3(9) - 12(3) + 1 = 54 - 27 - 36 + 1 = -8
\end{align*}
\]

2. (5 points) Evaluate \( \lim_{x \to -\infty} \frac{3x^2 - x + 4}{2x^2 + 5x - 8} \).

\[
\lim_{x \to -\infty} \frac{3x^2 - x + 4}{2x^2 + 5x - 8} = \lim_{x \to -\infty} \frac{3 - \frac{1}{x} + \frac{4}{x^2}}{2 - \frac{5}{x} - \frac{8}{x^2}} = \frac{3}{2}.
\]
3. (10 points) Let \( f(x) = (x+1)^5 - 5x - 2 \).

(a) Find the intervals of increase or decrease.
(b) Find the local maximum and minimum values.
(c) Find the intervals of concavity and the inflection points.
(d) Use parts (a) - (c) to sketch the graph.

\[
f'(x) = 5(x+1)^4 - 5 = 0 \implies (x+1)^4 = 1
\]

\[
\implies x + 1 = \pm \sqrt[4]{1} \quad \text{i.e.,} \quad |x+1| = 1
\]

\[
\implies x = 0 \quad \text{or} \quad x = -2
\]

Critical points: \( x = 0 \) and \( x = -2 \)

\[
\begin{array}{c|c|c|c}
\text{Critical Points} & + & - & + \\
\hline
-2 & 0 &
\end{array}
\]

\[
f''(x) = 5 \left( (x+1)^4 - 1 \right)
\]

\[
f''(-2) = 5 \left( (-1)^4 - 1 \right) > 0
\]

\[
f''(-1) = 5 \left[ -1 \right] < 0
\]

\[
f''(1) = 5 \left( 1^4 - 1 \right) > 0
\]

(a) \( f \) is increasing on \((-\infty, -2)\) and \((0, \infty)\)

(b) By the 1st derivative test, \( f(-2) = -1 + 10 - 2 = 7 \) is a local maximum and \( f(0) = 1 - 2 = -1 \) is a local minimum.

c) \( f''(x) = 20(x+1)^3 = 0 \implies x = -1 \)

\[
f''(-2) = 20(-1)^3 < 0
\]

\[
f''(0) = 20(1) > 0
\]

So \( f \) is concave down on \((-\infty, -1)\) and concave up on \((-1, \infty)\). \( x = -1 \) is an inflection point.

\[
\begin{array}{c|c|c|c}
\text{Critical Points} & + & - & + \\
\hline
-2 & -1 & 0
\end{array}
\]

Recall:
\[
f(-2) = 7
\]

\[
f(0) = -1
\]

\[
f(-1) = 5 - 2 = 3
\]