1. Show that if $f$ is an odd function, then $f(0) = 0$.

Proof. Suppose that $f$ is odd. Then $f(-x) = -f(x)$ for all $x$, so in particular, $f(0) = -f(0)$. The only number which is its own negative is 0, so $f(0) = 0$.

2. An apartment building has 100 units, and the rent is the same for every unit. If the rent is $800/month, every room will be rented, and for each $10 increase in the rent, one unit will be vacated. Write a function that represents the total revenue (rent $\times$ # of occupied units) that incorporates the above information.

Solution. Let $x$ be the number of times the rent has been raised by $10$, or equivalently, the number of apartments that have been vacated. Then the rent is given by $800 + 10x$ and the number of units is given by $100 - x$. So the total revenue is $(800 + 10x)(100 - x)$.

3. State whether each of the following equations defines a function in $x$ or not.

(a) $x + y^2 = 3$
(b) $y^3 = x$
(c) $\sin(x) = 7$

Solution. (a) This equation does not define a function in $x$. For example, $x = -1$ has two corresponding $y$-values: 2 and $-2$.
(b) This equation does define an equation because we can solve for $y$ to get $y = \sqrt[3]{x}$ and every number has a unique cube root.
(c) This equation does not define a function for two reasons: 1) the domain is empty, 2) even if it were not, $y$ has no dependence on $x$, so any $x$ in the domain would correspond to infinitely many $y$.

4. Find the range of $f(x) = 1 + 2 \cos x$.

Solution. Since $\cos x$ has range $[-1, 1]$, $1 + 2 \cos x$ has range $[-1, 3]$ (its lowest point occurs when $\cos x = -1$, so $f(x) = 1 - 2 = -1$, and its highest point occurs when $\cos x = 1$, so $f(x) = 1 + 2 = 3$).

5. How many roots does $\sin(x/5)$ have in the interval $[0, 2\pi]$?

Solution. $\sin(x/5)$ has period $2\pi/(1/5) = 10\pi$, so its roots starting at $x = 0$ are $0, 5\pi, 10\pi$, etc. So $\sin(x/5)$ has only the root $x = 0$ in the interval $[0, 2\pi]$.

6. State whether each of the following functions is one-to-one on the given interval or not.
(a) \( f(x) = \cos x \) on \([-\pi/2, \pi/2]\)
(b) \( f(x) = (x + 1)^2 \) on \([-1, 3]\)
(c) \( f(x) = x^5 \) on \(\mathbb{R}\)

Solution.  (a) This function is not one-to-one on that interval. For example, \( \cos(-\pi/2) = \cos(\pi/2) = 0 \).
(b) This function is one-to-one because the interval \([-1, 3]\) is entirely in the right half of the parabola, since the vertex occurs at \(x = -1\). (This is easier to see if graphed)
(c) This function is one-to-one because we can solve for \(x\) to get \(x = \sqrt[5]{y}\), and every number has a unique 5th root.

7. Find \( \cos(\arcsin(5/12)) \).

Solution. Let \( \theta = \arcsin(5/12) \), equivalently, \( \sin(\theta) = 5/12 \). We can build a right triangle with an angle \( \theta \) satisfying this, where 12 is the length of the hypotenuse. Then by the Pythagorean theorem, the square of the length of the third side is \(12^2 - 5^2 = 144 - 25 = 119\). So the third side length is \(\sqrt{119}\), so \(\cos(\theta) = \sqrt{119}/12\).

8. Solve the following equations for \(x\).

(a) \( e^{x^2+2x} = e^{-1} \)
(b) \( e^{2x} + 4e^x = -3 \)  (Hint: \( e^{2x} = (e^x)^2 \))
(c) \( \ln(x^2 - x) - \ln(x) = 2 \)

Solution.  (a) Taking \( \ln \) of both sides we get \( x^2 + 2x = -1 \), or \( x^2 + 2x + 1 = 0 \). Factoring we get \((x+1)^2 = 0\), so \(x = -1\).
(b) Easy solution: Notice that \( e^{2x} + 4e^x \) is always positive, so there is no \(x\) such that it will equal \(-3\). More involved solution: Rewrite the equation as \((e^x)^2 + 4e^x + 3 = 0\). Let \( u = e^x \). Then we can rewrite this as \(u^2 + 4u + 3 = 0\). Factoring we have \((u+3)(u+1) = 0\), so \(u = -3\) or \(u = -1\), or alternatively, \(e^x = -3\) or \(e^x = -1\). Again we can stop here and say that there is no such \(x\) since \(e^x\) is always positive, but let’s finish it up. \(e^x = -3\) means \(\ln(-3) = x\) and \(e^x = -1\) means \(\ln(-1) = x\). There is no such \(x\) since \(-1\) and \(-3\) are not in the domain of \(\ln x\).
(c) We can rewrite the left hand side as \( \ln\left(\frac{e^{2x} - x}{e^x}\right) = \ln(x - 1)\), and \(\ln(x - 1) = 2\) means \(e^2 = x - 1\). So \(x = e^2 + 1\).