Math 111 - Homework 3

Instructions: Although it is not stated after each question, you must explain your answers and show all of your work. You are allowed to work with others, but should only consult your textbook and your notes as resources, and of course you may consult me as well. Also, please try your best to write neatly.

This assignment is due on Wednesday October 2, 2013 by 5:00pm. You may turn it into me during class, or drop it off in my mailbox which is located in the mail room next to the front desk area on the 4th floor. I will also accept it via e-mail if you so desire.

1. Find all of the vertical and horizontal asymptotes of the given functions and the describe the behavior of the function near these asymptotes, i.e. give the left-hand and right-hand limits for each vertical asymptotes, and give the limits as $x \to \pm \infty$.

   (a) $f(x) = \frac{e^x}{e^x - 1}$
   
   (b) $f(x) = \frac{\sqrt[3]{x^6 + x^2 + 12}}{x^2 + 2x + 1}$
   
   (c) $f(x) = \arctan(\ln x)$

2. Find the following limits if they exist.

   (a) $\lim_{x \to \infty} \ln \left( \frac{x^2 + 2x + 2}{x^2 + 1} \right)$
   
   (b) $\lim_{x \to \infty} \frac{e^x - e^{-x}}{e^x + e^{-x}}$
   
   (c) $\lim_{x \to -\infty} \sqrt{\cos \left( \frac{\pi x + 1}{x + 2} \right)}$

3. Use the definition of the derivative to show that $f(x) = \begin{cases} x^2 \sin \left( \frac{1}{x} \right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0. \end{cases}$ is differentiable at $x = 0$. You may use Homework 2 problem 6 to compute the limit you get in the end.

4. Use the definition of the derivative to prove that the derivative of $f(x) = ax^2 + bx + c$ is $2ax + b$. (Yes, it will be tedious, but then you will feel like you’ve accomplished something at the end!)

   Note: You may now use this formula to find the derivatives in the following two problems. (Remember that if $a = 0$, the equation becomes $bx + c$, which is a line, but you should know how to find the slope of a line anyway.)

5. Let $f(x) = \begin{cases} x^2, & \text{if } x < 1 \\ 3x - 2, & \text{if } x \geq 1 \end{cases}$

   Show that $f$ is continuous but not differentiable at $x = 1$ first by graphing it, and then by using the limit definition of the derivative.
6. Let $f(x) = x^2 + x + 1$. Find where $f$ has a horizontal tangent line and give the equation of the line.

7. If $f(x) = x^{1/3}$, show that $f'(0)$ does not exist by using the definition of the derivative. What kind of failure of differentiability is this?