1. Show that $\frac{\sin \theta}{2} \leq \frac{\theta}{2}$ for $\theta$ in the interval $[0, \pi/2)$ using the diagram above. Hint: Use the fact that the area of the smaller triangle (OAB) is clearly smaller than the area of the sector (OAB). The area of a triangle is given by $\frac{1}{2}$ base $\times$ height and the area of a sector with angle $\theta$ is given by $\frac{r^2\theta}{2}$, where $r$ is the radius of the circle.

2. Show that $\frac{\theta}{2} \leq \frac{\tan \theta}{2}$ on $[0, \pi/2)$. Hint: Now use the fact that the sector (OAB) is clearly smaller than the bigger triangle (OAC).

3. Conclude that $\cos \theta \leq \frac{\sin \theta}{\theta} \leq 1$ on $(-\pi/2, \pi/2)$. (Note that we are deducing this from statements about area, so reflecting the diagram across the x-axis does not change the inequalities).

4. Use the conclusion above to show that $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$.

5. Show that $\lim_{\theta \to 0} \frac{1 - \cos \theta}{\theta} = 0$. Hint: Multiply the top and bottom by $1 + \cos \theta$. Then use the identity $\sin^2 \theta + \cos^2 \theta = 1$ and the previous result.

6. Prove that the derivative of $\sin x$ is $\cos x$. You will need the following formula:

$$\sin(A + B) = \sin A \cos B + \cos A \sin B.$$