Math 112 Worksheet 2: Improper Integrals

Instructions: Say whether each of the following improper integrals converges or diverges. If it converges, say what it converges to.

1. \[
\int_{0}^{\infty} \frac{1}{1 + x^2} \, dx
\]

2. \[
\int_{1}^{\infty} \frac{1}{x^2 + x} \, dx
\]

3. \[
\int_{-1}^{0} \frac{1}{x^2 + 2x + 1} \, dx
\]

4. \[
\int_{0}^{\infty} x \ln x \, dx \quad \text{(Hint: In how many ways is this improper?)}
\]

5. \[
\int_{-1}^{1} \frac{1}{\sqrt{x}} \, dx
\]

6. \[
\int_{-\infty}^{\infty} e^{-|x|} \, dx
\]

7. \[
\int_{0}^{\pi/2} \tan x \, dx
\]

8. Challenge problem: \[
\int_{0}^{\infty} x^{1000} e^{-x} \, dx
\]
Solutions

1. Converges to $\pi/2$
2. Converges to $\log 2$
3. Diverges
4. Diverges
5. Converges to 3
6. Converges to 2
7. Diverges
8. Converges to $1000!$ (that’s a factorial, not a sign of excitement, although it’s exciting!) Note: If you can do this, you basically proved a theorem where we replace 1000 by any $n \geq 0$. 