Since the original problem 2 on the improper integrals worksheet involves a lot of precision, I will write up the solution. I will never assign a problem like this (on purpose), but it’s good to see.

**Exercise:** Does the integral \( \int_{-\infty}^{\infty} \frac{1}{x^2 + x} \, dx \) converge or diverge?

**Solution:** First we use the method of partial fractions to write \( \frac{1}{x^2 + x} \) as \( \frac{1}{x} - \frac{1}{x + 1} \). So we now have to evaluate the integral

\[
\int_{-\infty}^{\infty} \left( \frac{1}{x} - \frac{1}{x + 1} \right) \, dx.
\]

The integrand is undefined at \( x = 0 \) and at \( x = -1 \), and we also have to deal with the infinite limits, so we will split up the integral as follows:

\[
\int_{-\infty}^{-2} \left( \frac{1}{x} - \frac{1}{x + 1} \right) \, dx + \int_{-2}^{-1} \left( \frac{1}{x} - \frac{1}{x + 1} \right) \, dx + \int_{-1}^{-1/2} \left( \frac{1}{x} - \frac{1}{x + 1} \right) \, dx + \int_{0}^{1/2} \left( \frac{1}{x} - \frac{1}{x + 1} \right) \, dx + \int_{1/2}^{1} \left( \frac{1}{x} - \frac{1}{x + 1} \right) \, dx + \int_{1}^{\infty} \left( \frac{1}{x} - \frac{1}{x + 1} \right) \, dx
\]

so that each integral is improper at only one point on its interval. Yes, this looks insane, but remember, all you need for an improper integral of this form to diverge is for ONE of those integrals above to diverge. This easiest way to see that this will diverge is to notice that the antiderivative of the integrand is \( \ln \left| \frac{x}{x + 1} \right| \) after using log rules. This will be undefined if we plug in anything negative, so all of the integrals that are over intervals of negative numbers will give us an undefined limit. Therefore the entire integral is said to diverge.