Trigonometry Review

First, starting with the very basics, we should all know SOHCAHTOA.

\[
\sin x = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cos x = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \tan x = \frac{\text{opposite}}{\text{adjacent}}
\]

\[
\csc x = \frac{1}{\sin x} \quad \sec x = \frac{1}{\cos x} \quad \cot x = \frac{1}{\tan x}
\]

We should also know what the graphs of the trigonometric functions look like.

Graph of \( y = \sin x \)

Graph of \( y = \cos x \)

Graph of \( y = \tan x \)
Now let’s take a look at how to evaluate a trigonometric function. In Calculus, we always use radians instead of degrees. However, you will still need to know that $\frac{\pi}{2}$ forms a 90 degree angle, $\pi$ forms a 180 degree angle, etc.

Consider the unit circle (i.e. a circle with a radius of 1, centered at the origin).

We know the circumference of the circle is $2\pi$. So, 1/4th of the circle will have length of $\pi/2$. From here, we can determine where $\pi/6$, $\pi/4$, and $\pi/3$ are. These numbers represent the distance from 0 in the counterclockwise direction.

We will use this unit circle to help us evaluate trigonometric functions. Note that:

\[
\begin{align*}
\sin x &= y \text{- coordinate} \\
\cos x &= x \text{- coordinate} \\
\tan x &= \frac{\sin x}{\cos x}
\end{align*}
\]
Whether the value is positive or negative depends on which quadrant we are in:

<table>
<thead>
<tr>
<th>Quadrant</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values in the Quadrant</td>
<td>$[0, \frac{\pi}{2})$</td>
<td>$[\frac{\pi}{2}, \pi)$</td>
<td>$[\pi, \frac{3\pi}{2})$</td>
<td>$(\frac{3\pi}{2}, 2\pi)$</td>
</tr>
<tr>
<td>$x$ (cosine)</td>
<td>positive</td>
<td>negative</td>
<td>negative</td>
<td>positive</td>
</tr>
<tr>
<td>$y$ (sine)</td>
<td>positive</td>
<td>positive</td>
<td>negative</td>
<td>negative</td>
</tr>
</tbody>
</table>

**Ex:** Evaluate $\sin \theta$, $\cos \theta$, and $\tan \theta$, where $\theta$ is:

(a) $\frac{2\pi}{3}$  
(b) $\frac{5\pi}{4}$  
(c) $\frac{11\pi}{6}$

If you are given a value of $\theta$ that is not in $[0, 2\pi]$, then you need to keep adding or subtracting $2\pi$ until you get a value inside this interval.
Ex: Evaluate sin $\theta$, cos $\theta$, and tan $\theta$, where $\theta$ is:

(a) $\frac{7\pi}{3}$ \hspace{1cm} (b) $\frac{23\pi}{4}$ \hspace{1cm} (c) $-\frac{11\pi}{6}$

More practice: Evaluate the following.

<table>
<thead>
<tr>
<th></th>
<th>$3\pi$</th>
<th>$\frac{5\pi}{6}$</th>
<th>$-\frac{\pi}{4}$</th>
<th>$-\frac{2\pi}{3}$</th>
<th>$\frac{3\pi}{2}$</th>
<th>$-2\pi$</th>
<th>$-\frac{3\pi}{2}$</th>
<th>$\frac{7\pi}{6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>sin $\theta$</td>
<td></td>
<td></td>
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<tr>
<td>cos $\theta$</td>
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<td>tan $\theta$</td>
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