André-Louis Cholesky: Mathematician, Topographer and Army Officer

by Claude Brezinski and Dominique Tournès

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In the early morning of 31 August 1918, in a quarry near the village of Bagneux in the region of Picardy, a German artillery shell ended the life of André-Louis Cholesky, a 42-year-old French army officer. Within a few weeks, the Allies would break through the Hindenburg Line and World War I would be ended with the signing of the armistice on November 11. A few years later, in 1924, the *Bulletin Géodésique* published a paper by a "Cdt. Benoît," titled *Note sur une méthode de résolution des équations normales provenant de l'application de la méthode des moindre carrés à un système d'équations linéaires en nombre inférieur à celui des inconnues* (*Procédé du Commandant Cholesky*). Over time, the name of Cholesky would become familiar to numerical analysts, geodesists, statisticians, and eventually to virtually anyone making use of numerical methods in scientific work. Since the appearance of Benoît's paper, Cholesky's name has been eponymous with a simple and elegant algorithm for solving a broad class of systems of linear equations, including those arising from the treatment of observations using the method of least squares. The practical importance of Cholesky's method is due to the fact that large systems of linear equations with a symmetric and positive definite coefficient matrix are ubiquitous in computational science and engineering. Among other advantages, Cholesky's algorithm is much simpler and economical than several methods (both direct and iterative) that Gauss himself and others had devised for solving the same problem. It can be applied to any linear system with a symmetric and positive (semi-)definite coefficient matrix A, and it is based on the factorization of A into the product of a lower triangular matrix and its transpose. Hence, the method provides a systematic, efficient, and numerically reliable way to reduce a non-negative definite quadratic form to a diagonal form by means of a congruence transformation. The modern view of numerical linear algebra as being to a large extent the study and systematic use of matrix decompositions has certainly been influenced by Cholesky's posthumously published work. The usefulness of Cholesky's method can be quickly gauged by turning to Google: a search of the name "Cholesky" returns nearly half a million hits (as of August 2015). Entering phrases such as "Cholesky method," "Cholesky decomposition," or "Cholesky algorithm" also produces hundreds of thousands of documents. Even the common misspelling "Choleski" yields more than 50,000 hits.

In contrast to the widespread recognition accorded to his method, Cholesky's identity, and even the correct spelling of his name, have long remained somewhat mysterious. Information on his life circumstances was very scarce until approximately 10 years ago, and very few people, if any, knew whether he had made any other contributions to science. Even more obscure has been the figure of Commandant Benoît, the man responsible for bringing Cholesky's method into print.

Several years ago, Claude Brezinski, a well-known expert in the field of numerical analysis, set himself the task to discover the man and the story behind Cholesky's algorithm. This multiyear effort culminated in the book under review, coauthored with Dominique Tournès, a professional historian of science. It is the result of years of archival research, some detective work, and a good bit of luck. As it turns out, Cholesky's archive had been preserved by his family, and in 2003 his grandchildren contacted Brezinski for help in classifying the material after deciding to donate it to the *École Polytechnique*.

The book by Brezinski and Tournès finally lifts the veil of mystery about Cholesky, his life and military career, and his scientific contributions. We also learn the identity of Commandant Benoît, and about a few other characters whose life and work played an important role in Cholesky's life. The book, however, is much more than an account of the life and scientific work of Cholesky. Besides the necessary historical context, Brezinski and Tournès provide background material about other topics, including topography, nomography and other methods for graphical calculations, and a brief historical account of methods for solving systems of linear equations. The book also contains some information on the training of engineers in various French military schools and even in correspondence courses in the late XIX and early XX centuries. Furthermore, the book presents some very interesting hitherto unpublished manuscripts, some of which are given both in the original French and in English translation. All told, these manuscripts and their analyses occupy a little less than half of the volume. These documents, which were up to now virtually inaccessible, are an integral part of the work, and the authors deserve thanks for making them available. The volume also contains a large number of illustrations and reproductions of black-and-white photographs, including many portraits of Cholesky throughout the years.

The first chapter is a biography of Cholesky. We learn a little about his infancy and studies, and a good deal about his work as a topographer in France, Crete, and North Africa, and about his military service in World War I, including a stint in Romania. The chapter ends with a gripping account of Cholesky's last days and untimely death in the Second Battle of Picardy, including a first-hand

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account by some of his comrades. A few pages are devoted to his activities from 1909 to 1914 as a teacher in the correspondence courses of the *École Spéciale des Travaux Publics, du Bâtiment et de l'Industrie* (ESTP). The interesting story of this school and its founder, Léon Eyrolles, is told in greater detail in Chapter 6.

Chapter 2 is devoted to an account of Cholesky's ancestry going back to the XVIII Century, his immediate family, and his descendants down to the present day. The chapter contains a great deal of minute information regarding Cholesky's family, including his relation to the family of the former Shah of Iran. In this reviewer's opinion, much of this material is of limited interest to most readers and could have been left out without detriment.

Chapter 3 contains a brief introduction to the field of topography to the extent required to grasp Cholesky's main contribution to this subject, known as the method of double-run leveling, which became widely adopted.

Chapter 4 is, at least to this reviewer, the most interesting part of the book. It discusses Cholesky's method for solving linear systems. The chapter opens with a short history of the method of least squares and its gradual adoption by astronomers and geodesists. Besides the often-told circumstances of Gauss's determination of the orbit of Ceres based on the observations of Giuseppe Piazzi and the unfortunate ensuing priority dispute between Legendre and Gauss, we also learn about important earlier contributions by Ruggero Boscovich, Johann Tobias Mayer, and Pierre-Simon de Laplace to the problem of determining the orbit of a celestial body from observations. Mention is made of Gauss's work on the probabilistic foundations of least squares and of Markov's later contributions. The chapter also contains a valuable description of how the method of least squares is applied to the adjustment of geodetic networks, the problem that led Cholesky to the development of his method. Several pages are devoted to a brief history of methods for solving linear systems up to the time of Cholesky, limited to the class of *direct methods* (i.e., methods that are guaranteed to return an exact solution in a finite number of steps, provided that the system is consistent and that no rounding errors occur). These include the so-called Gaussian elimination, and the variants introduced by Wilhelm Jordan (a German surveyor), Myrick Hascall Doolittle (an American who had studied at Harvard under Benjamin Pierce), and Tadeusz Banachiewicz (a Polish astronomer), among others. Many references are given here to both the primary and secondary sources, the latter including the recent comprehensive studies by Joseph Grcar on the history of elimination methods for solving linear systems. Although we do not know exactly when Cholesky developed his algorithm, there seems to be little doubt that he deserves priority over other authors (a rare exception to Arnold's Principle). As the authors note (p. 91), Otto Toeplitz published in 1907 a proof of the fact that every Hermitian positive definite matrix can be factored into the product LL^* with L lower triangular, but he did not provide an algorithm for computing the entries of L. Instead, Toeplitz provided explicit expressions, involving determinants, for the entries of L^{-1}. In contrast with Cholesky's compact algorithm, these formulas do not have much practical value. The core of the chapter (and, arguably, the book) consists of an English translation of an 8-page unpublished manuscript by Cholesky describing the algorithm. The manuscript, which was among the papers possessed by Cholesky's family, is dated 2 December 1910. It appears to have been written several years after Cholesky's first use of the method in his fieldwork as a surveyor. The rest of the chapter contains a detailed analysis of the manuscript from a modern numerical analyst's perspective, and an account of the subsequent diffusion and influence of Cholesky's method. As the authors explain, the adoption of Cholesky's method owes not a little to the publicity given to it shortly after the end of World War II by British mathematicians and computer pioneers, including Alan Turing, Leslie Fox, Jim Wilkinson, and especially John Todd.

Chapter 5 concerns other documents, manuscripts, and published works of Cholesky, including his fairly successful *Cours de Topographie* and notes prepared for the students at ESTP.

Another highlight of the book is Chapter 7, which deals with graphical computations (including a concise history of the subject) and an unfinished manuscript by Cholesky on the topic. This appears to have been part of an intended textbook, the completion of which was unfortunately interrupted by the onset of World War I. This material should be of considerable interest to historians of computing methods.

Chapter 8 consists of a brief biography of Ernest Benoît. Chapter 9 contains translations of military documents related to Cholesky's service in the Army, notebooks of Cholesky, and a description of the contents of the *Fonds Cholesky* at the *École Polytechnique*. Cholesky's journal entries from his notebooks give the reader an idea of his life while he was participating in a survey of France's *Massif Central* in 1905–1906. Although brief, they make for some interesting and occasionally amusing reading. The volume concludes with a few appendices, including the French originals of Cholesky's manuscript on the solution of systems of linear equations and of the draft of his book on graphical calculations. An extensive bibliography completes the book.

The volume represents a valuable contribution to the growing literature on the history of applied and computational mathematics, a discipline that has long suffered from neglect on the part of professional historians of science. It must be mentioned, however, that the book is not without shortcomings, many of which could have been addressed had the manuscript been reviewed by a competent editor. (The authors gratefully acknowledge the assistance of a copy editor, but there is little evidence of any serious copyediting in the end product.) The quality of writing and exposition is unequal, perhaps reflecting the dual authorship. The English is often poor, sometimes to the point of unreadability. This is especially true when it comes to translations of nontechnical documents from the original French; readers not equipped with some knowledge of French grammar and sentence structure may be left wondering about the meaning of some passages. Subject–verb agreement errors occur frequently, as in this sentence on

page 60: "Iran, it changes the atmosphere, especially at that time. We live at the bottom of a large shady park, the park Amine Dawlé, and in the evenings, my mother go and join her husband, secretly in the palace." Stray French words frequently appear interspersed in the English text. For example: "On 13 avril 1816, King Louis XVIII..." (p. 4). On page 38, one reads: "...her right wing form a tenaille with the Aisne"; note that "her" refers to the 10th Army, and that two lines above, the authors write "its left wing." Also, "there was a billard room" (p. 142); "the baggage of correspondence education" (p. 144); and other instances of "false friends." In one place, the term "geometer" is used with the meaning of "surveyor." Military ranks sometimes are given in English, but more often in French. For example, "Cholesky is appointed lieutenant en second at the..." (p. 7); "...and will become a général..." (p. 8 and elsewhere); "Cholesky was promoted to premier lieutenant" (p. 10), and so forth. This is probably intentional, but there are many inconsistencies (other French words or phrases are usually italicized) and the result is that the writing comes across as sloppy. Also, the authors' habit of giving, for almost all named people, no matter how secondary to the main narrative, the places and dates of birth and death makes for rather heavy reading. This information should have been limited to the main characters.

The mathematical discussion is occasionally marred by mistakes: several incorrect or confusing statements occur in the account of the diffusion of Cholesky's method on page 112. On page 120 we read of "a lower diagonal matrix multiplied by its transpose" (presumably, the authors meant to write "lower triangular"). Surprisingly, in the discussion of incomplete Cholesky factorizations, one of the most important modern offsprings of Cholesky's method, credit is given to Kershaw, but not to Meijerink and van der Vorst, who introduced the technique (p. 121).

Although these deficiencies are altogether minor, they are unfortunate because they subtract from the enjoyment of an otherwise interesting and valuable work of scholarship. Had the publisher and authors taken the responsibilities of copyediting and proofreading more seriously, the end result would have been more in line with the high professional standards that readers have come to expect of them.

In summary, the book will be of interest to applied mathematicians with a curiosity about historical matters, and to professional historians of science who specialize in the history of "practical mathematics," topography, geodesy, and computing methods.

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