Cor. v. And if the globes move in different mediums, the space, in a medium which, other things being equal, resists the most, must be diminished in the ratio of the greater resistance. For the time (by this Proposition) will be diminished in the ratio of the augmented resistance, and the space in the ratio of the time.

**Lemma 2**

The moment of any genitum is equal to the moments of each of the generating sides multiplied by the indices of the powers of those sides, and by their coefficients continually.

I call any quantity a *genitum* which is not made by addition or subtraction of divers parts, but is generated or produced in arithmetic by the multiplication, division, or extraction of the root of any terms whatsoever; in geometry by the finding of contents and sides, or of the extremes and means of proportionals. Quantities of this kind are products, quotients, roots, rectangles, squares, cubes, square and cubic sides, and the like. These quantities I here consider as variable and indetermined, and increasing or decreasing, as it were, by a continual motion or flux; and I understand their momentary increments or decrements by the name of moments; so that the increments may be esteemed as added or affirmative moments; and the decrements as subtracted or negative ones. But take care not to look upon finite particles as such. Finite particles are not moments, but the very quantities generated by the moments. We are to conceive them as the just nascent principles of finite magnitudes. Nor do we in this Lemma regard the magnitude of the moments, but their first proportion, as nascent. It will be the same thing, if, instead of moments, we use either the velocities of the increments and decrements (which may also be called the motions, mutations, and fluxions of quantities), or any finite quantities proportional to those velocities. The coefficient of any generating side is the quantity which arises by applying the *genitum* to that side.

Wherefore the sense of the Lemma is, that if the moments of any quantities $A$, $B$, $C$, &c., increasing or decreasing by a continual flux, or the velocities of the mutations which are proportional to them, be called $a$, $b$, $c$, &c., the moment or mutation of the generated rectangle $AB$ will be $ab + ba$; the moment of the generated content $ABC$ will be $aBC + bAC + cAB$; and the moments of the generated powers $A^2$, $A^3$, $A^4$, $A^{1/2}$, $A^{2/3}$, $A^{1/3}$, $A^{2/2}$, $A^{-1}$, $A^{-2}$, $A^{-1/2}$ will be $2aA$, $3aA^2$, $4aA^3$, $\frac{1}{2}aA^{1/2}$, $\frac{1}{3}aA^{1/3}$, $\frac{1}{2}aA^{-1/2}$, $\frac{1}{3}aA^{-1/3}$, $-aA^{-2}$, $-2aA^{-3}$, $-\frac{1}{2}aA^{-3/2}$ respectively; and, in general, that the moment of any power $A^n$ will be \( \frac{n}{m} aA^{n-1} \). Also, that the moment of the generated quantity $AB$ will be $2ab + ba^2$; the moment of the generated quantity $AB^2C$ will be $3aA^2BC + 4bA^2BC + 2cA^2BC$; and the moment of the generated quantity $\frac{A^2}{B^2}$ or $A^2B^{-2}$ will be $3aA^2B^{-2} - 2abA^2B^{-3}$; and so on. The Lemma is thus demonstrated.

Case 1. Any rectangle, as $AB$, augmented by a continual flux, when, as yet, there wanted of the sides $A$ and $B$ half their moments $\frac{1}{2}a$ and $\frac{1}{2}b$, was $A - \frac{1}{2}a$ into $B - \frac{1}{2}b$, or $AB - \frac{1}{2}a B - \frac{1}{2}b A + \frac{1}{4}ab$; but as soon as the sides $A$ and $B$ are augmented by the other half-moments, the rectangle becomes $A + \frac{1}{2}a$ into $B + \frac{1}{2}b$, or $AB + \frac{1}{2}a B + \frac{1}{2}b A + \frac{1}{4}ab$. From this rectangle subtract the former rectangle, and there will remain the excess $ab + ba$. There-
fore with the whole increments $a$ and $b$ of the sides, the increment $ab + ba$ of the rectangle is generated.

**Case 2.** Suppose $AB$ always equal to $G$, and then the moment of the content $ABC$ or $GC$ (by Case 1) will be $\frac{aC + cG}{2G}$, that is (putting $AB$ and $ab + ba$ for $G$ and $g$), $aBC + bAC + cAB$. And the reasoning is the same for contents under ever so many sides.

**Case 3.** Suppose the sides $A$, $B$, and $C$, to be always equal among themselves; and the moment $ab + ba$, of $A^2$, that is, of the rectangle $AB$, will be $2aA$; and the moment $aBC + bAC + cAB$ of $A^2$, that is, of the content $ABC$, will be $3aA^2$. And by the same reasoning the moment of any power $A^n$ is $naA^{n-1}$.

**Case 4.** Therefore since $\frac{1}{A}$ into $A$ is 1, the moment of $\frac{1}{A}$ multiplied by $A$, together with $\frac{1}{A}$ multiplied by $a$, will be the moment of 1, that is, nothing. Therefore the moment of $\frac{1}{A}$, or of $A^{-1}$, is $-\frac{a}{A^2}$. And generally since $\frac{1}{A^n}$ into $A^n$ is 1, the moment of $\frac{1}{A^n}$ multiplied by $A^n$ together with $\frac{1}{A^n}$ into $naA^{n-1}$ will be nothing. And, therefore, the moment of $\frac{1}{A^n}$ or $A^{-n}$ will be $-\frac{na}{A^{n+1}}$.

**Case 5.** And since $A^{1/2}$ into $A^{1/2}$ is $A$, the moment of $A^{1/2}$ multiplied by $2A^{1/2}$ will be $a$ (by Case 3); and, therefore, the moment of $A^{1/2}$ will be $\frac{a}{2A^{1/2}}$ or $\frac{1}{2}aA^{-1/2}$. And generally, putting $A^m$ equal to $B$, then $A^n$ will be equal to $B^m$, and therefore $maA^{m-1}$ equal to $nB^{n-1}$, and $maA^{-1}$ equal to $nB^{-1}$, or $nbA^{-n}$; and therefore $\frac{n}{m}A^{-n}$ is equal to $b$, that is, equal to the moment of $A^{-n}$.

**Case 6.** Therefore the moment of any generated quantity $A^mB^n$ is the moment of $A^m$ multiplied by $B^n$, together with the moment of $B^n$ multiplied by $A^m$, that is, $maA^{m-1}B^n + nbB^{n-1}A^m$; and that whether the indices $m$ and $n$ of the powers be whole numbers or fractions, affirmative or negative. And the reasoning is the same for higher powers.

**Cor. 1.** Hence in quantities continually proportional, if one term is given, the moments of the rest of the terms will be as the same terms multiplied by the number of intervals between them and the given term. Let $A$, $B$, $C$, $D$, $E$, $F$ be continually proportional; then if the term $C$ is given, the moments of the rest of the terms will be among themselves as $-2A$, $-B$, $D$, $2E$, $3F$.

**Cor. 2.** And if in four proportionals the two means are given, the moments of the extremes will be as those extremes. The same is to be understood of the sides of any given rectangle.

**Cor. 3.** And if the sum or difference of two squares is given, the moments of the sides will be inversely as the sides.

**Scholium**

In a letter of mine to Mr. J. Collins, dated December 10, 1672, having described a method of tangents, which I suspected to be the same with Sluse's method, which at that time was not made public, I added these words: *This is one particular, or rather a Corollary, of a general method, which extends itself,*
without any troublesome calculation, not only to the drawing of tangents to any curved lines, whether geometrical or mechanical or in any manner respecting right lines or other curves, but also to the resolving other abstruser kinds of problems about the crookedness, areas, lengths, centres of gravity of curves, &c.; nor is it (as Hudden's method de maximis et minimis) limited to equations which are free from surd quantities. This method I have interwoven with that other of working in equations, by reducing them to infinite series. So far that letter. And these last words relate to a treatise I composed on that subject in the year 1671. The foundation of that general method is contained in the preceding Lemma.

Proposition 8. Theorem 6

If a body in an uniform medium, being uniformly acted upon by the force of gravity, ascends or descends in a right line; and the whole space described be divided into equal parts, and in the beginning of each of the parts (by adding or subtracting the resisting force of the medium to or from the force of gravity, when the body ascends or descends) you derive the absolute forces: I say, that those absolute forces are in a geometrical progression.

Let the force of gravity be represented by the given line AC; the force of resistance by the indefinite line AK; the absolute force in the descent of the body by the difference KC; the velocity of the body by a line AP, which shall be a mean proportional between AK and AC, and therefore as the square root of the resistance; the increment of the resistance made in a given interval of time by the short line KL, and the contemporaneous increment of the velocity by the short line PQ; and with the centre C, and rectangular asymptotes CA, CH, describe any hyperbola BNS meeting the erected perpendiculars AB, KN, LO in B, N, and O. Because AK is as AP², the moment KL of the one will be as the moment 2AP·PQ of the other, that is, as AP·KC; for the increment PQ of the velocity is (by Law n) proportional to the generating force KC. Let the ratio of KL be multiplied by the ratio KN, and the rectangle KL·KN will become as AP·KC·KN; that is (because the rectangle KC·KN is given), as AP. But the ultimate ratio of the hyperbolic area KNOL to the rectangle KL·KN becomes, when the points K and L coincide, the ratio of equality. Therefore that hyperbolic evanescent area is as AP. Therefore the whole hyperbolic area ABOL is composed of intervals KNOL which are always proportional to the velocity AP; and therefore is itself proportional to the space described with that velocity. Let that area be now divided into equal parts, as ABMI, IMNK, KNOL, &c., and the absolute forces AC, IC, KC, LC, &c., will be in a geometrical progression. Q.E.D. And by a like reasoning, in the ascent of the body, taking, on the contrary side of the point A, the equal areas ABmi, imnk, knol, &c., it will appear that the absolute forces AC, iC, kC, IC, &c., are continually proportional. Therefore if all the spaces in the ascent and descent are taken equal, all the absolute forces IC, kC, iC, AC, IC, KC, LC, &c., will be continually proportional.

Cor. 1. Hence if the space described be represented by the hyperbolic area ABNK, the force of gravity, the velocity of the body, and the resistance of the