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# Francesco Paolo Cantelli

b. 20 December 1875
d. 21 July 1966


## Summary

**Francesco Paolo Cantelli made fundamental contributions to the foundations of probability theory and to the clarification of different types of probabilistic convergence. He is remembered through the Borel–Cantelli Lemma, the Glivenko–Cantelli Theorem, and Cantelli's Inequality. He also made seminal contributions to actuarial science.**


Cantelli was born in Palermo, Sicily. He attended the local university where he graduated in 1899 in mathematics with a thesis on celestial mechanics. Cantelli's advisor, Filippo Angelitti, was Director of the Palermo Astronomical Observatory, and he began working as Angelitti's assistant while still a student. Cantelli remained at the Observatory until 1903. His first publications were about various topics in astronomy and celestial mechanics, including a remarkable study (Cantelli, 1958, 1–45) of the positions of celestial bodies as described by Dante Alighieri in the *Divina Commedia*, a favorite topic of Angelitti's. Based on this study, Cantelli was able to conclude that Dante's masterpiece was probably written in the year 1301 rather than in 1300 as claimed by some commentators. During this formative period Cantelli became interested in probability and statistics, the knowledge of which is indispensable for dealing with observational errors in astronomy.

In 1903 Cantelli successfully competed for a position as an actuary with one of the major banks in Rome, where he remained for the next 20 years. During this period, despite intense professional activity, Cantelli managed to remain active in research, publishing a number of important papers on probability, mathematical statistics, financial mathematics, and mathematical economics. He also maintained close contacts with members of the mathematical community at the University of Rome, particularly with Guido Castelnuovo (1865–1952), in an effort to encourage mathematicians' interest in the study of probability theory, which Cantelli regarded as a branch of mathematical analysis. This point of view was not widely held during the early years of the 20th century.

In 1922 Cantelli obtained the first (and, for many years, the only) *libera docenza* in probability theory from the University of Rome, and in 1923 he was awarded a full professorship in Financial Mathematics at the University of Catania. In 1925 he moved to the University of Naples, and in 1931 back to Rome as a professor in the *Istituto Superiore di Scienze Economiche e Commerciali*, later to become the *Facoltà di Economia e Commercio* of the University of Rome. He retired from teaching and became Emeritus Professor in 1951.

Among Cantelli's awards was the prize for the calculus of probability and its applications, sponsored by the Milan Insurance Company, from the *Accademia Nazionale dei Lincei*. In due course he was elected a member of the *Accademia Nazionale dei Lincei* and of several other academies. Cantelli served as chair or vice chair of a number of committees on applied mathematics within the *Consiglio Nazionale delle Ricerche*, and at some point held the presidency of the *Istituto Nazionale per le Applicazioni del Calcolo*, which was founded by Mauro Picone with Cantelli's support. He also served as an expert for various Italian ministries, and did extensive consulting work for the insurance industry. In respect of the political and institutional context, Cantelli exerted considerable influence in scientific, academic and corporate circles in Fascist Italy. However, he did not compromise himself with the regime to the point of experiencing difficulties after its fall during World War II. Details of Cantelli's career are given by Ottaviani (1966).

Cantelli's many former pupils and friends collected many of his most important papers in a volume (Cantelli, 1958), which was presented to him at a conference held in Rome in his honour. He died in Rome.

At the beginning of the 20th century the calculus of probability, despite its success in applications, was still regarded with suspicion by most mathematicians, largely due to the lack of generally accepted rigorous definitions for many of the fundamental notions, including that of probability itself. The classical work of Laplace (q.v.) was not immune from deficiencies and inconsistencies. Most mathematicians regarded the calculus of probability as an empirical science rather than a mathematical one. An important exception was the work of the St. Petersburg School, which included P.L. Chebyshev (q.v.), A.M. Liapunov and A.A. Markov (q.v.). These mathematicians made great contributions to the theory of probability, with an emphasis on rigorous proofs for the Law of Large Numbers and for the Central Limit Theorem. The work of the Russians, however, still lacked an axiomatic basis for probability.

Cantelli and Castelnuovo were among the first Italian mathematicians to take an active interest in the theory of probability. Cantelli published a first system of axioms for probability theory in 1905, using as a model the axiomatization of elementary Euclidean geometry given a few years earlier by G. Veronese. Shortly afterwards, but without knowledge of Cantelli's work, important contributions by Borel (q.v.) in 1909, F. Bernstein (q.v.) in 1911, Hausdorff in 1914, S.N. Bernstein (q.v.) in 1917, Fréchet (q.v.) in 1921, Slutsky (q.v.) in 1922, Lomnicki in 1923, and Steinhaus in 1923 appeared in print, a clear indication of the growing interest in the theory of probability among "mainstream" mathematicians. Borel and Hausdorff appear to be the first to have established a link between Lebesgue's theory of measure and integration (first published in 1902) and the theory of probability. Cantelli was to return to the topic with a different system of axioms in 1932 in his *Giornale* (see below), when he gave one of the first abstract treatments of probability theory using Lebesgue's measure theory. This was one year before the publication of Kolmogorov's celebrated and definitive work of 1933 (which cites Cantelli's work on foundations). See Cantelli (1958, pp. 289–297), and the discussion in Benzi (1988) and Regazzini (1998). Benzi (1995) also speaks of an early attempt at axiomatization using Lebesgue's measure theory by another Italian, Ugo Broggi (1880–1965) in a Göttingen dissertation of 1907.

In the years 1916–1917, while striving to give rigorous formulations and proofs of the Law of Large Numbers, Cantelli was naturally led to distinguish between different types of convergence for sequences of random variables. This required him to introduce a rigorous definition of random variable, which he was the first to give (see Cantelli, 1958, pp. 175–188). The notions of almost certain (or "strong") convergence for a sequence of random variables and that of convergence in probability (or "weak" convergence) are clearly identified here for the first time. Cantelli went on to prove various versions of the Law of Large Numbers; many of his results would be improved in the years 1925–1935 by Slutsky, S.N. Bernstein, Khinchin, and Kolmogorov.

The result that is now known as the Borel–Cantelli Lemma relates to a sequence of events $\{A_n\}$ in probability space. Borel's zero-one law of 1909 states that if the events are independent, then if the sum of the $P(A_n)$ converges, the probability that an infinite number of events occurs is zero. If the sum diverges to infinity, the probability that infinitely many occur is unity. Cantelli (1917) showed that the hypothesis of independence could be dropped entirely from the *convergence* part by using Boole's (q.v.) Inequality: $P(A_1 A_2 \cdots A_r) \geq 1 - \sum_{k=1}^{r} P(\bar{A}_k)$, which does not require independence. Cantelli did this within his investigation of the Law of Large Numbers, where the successive averages $X_n$, $n \geq 1$, of independent random variables are clearly statistically dependent, as, consequently, will be events $\{A_n\}$ defined in terms of them. Apparently, Borel's work was not noticed by other probabilists until a footnote in a paper of Slutsky (1928) mentioned Cantelli in a secondary role. The matter was noticed by Cantelli's compatriots, and came to a head during the 8th International Congress of Mathematicians, held in Bologna, Italy, 3–10 September, 1928, which Cantelli and Slutsky attended, within a galaxy of established and coming stars of probability and statistics. An account of Cantelli's (1917) paper and of the *Congresso* dispute and its aftermath is contained in Seneta (1992). While there appears to be a serious flaw in Borel's reasoning, Cantelli's has no flaws, and does not exaggerate its conclusion.

Cantelli's (1928) elegant inequality for a random variable $X$ with mean and variance, respectively, $\mu, \sigma^2$, which also occurs in the context of the *Congresso*, states that

$$P(X - \mu \geq t\sigma) \leq \frac{1}{1 + t^2}, \quad t > 0.$$

Cantelli founded and edited the *Giornale dell'Istituto Italiano degli Attuari*, whose first volume appeared in 1930, and which served as a vehicle for contributions (generally published in Italian) in the 1930s by researchers of the stature of von Mises (q.v.), Kolmogorov, Khinchin, Slutsky, Romanovsky, Glivenko, and others. Both Cantelli and Castelnuovo, in this and other ways, disseminated knowledge of the work of the Russian probabilists among Western researchers.

Cantelli's international standing led to an invitation to teach a course at the prestigious Institute Poincaré in Paris in 1933. As one of the leading experts on the foundations of probability he did not hesitate to engage in vigorous debates on controversial aspects of this subject with other internationally recognized experts, such as Richard von Mises; see Benzi (1988, 1995) and Regazzini (1998). From an epistemological standpoint, Cantelli clarified his position on the foundations of probability in the course of a debate with von Mises, which appeared on the pages of his *Giornale* in 1936. As is well known, von Mises advocated a *frequentist* approach, albeit in the restricted setting provided by his theory of *collectives*. Cantelli, who had pointed out the contradictions inherent in the classical definition of probability as the limit of frequencies, insisted that no *interpretation* of probability is necessary within the abstract (measure–theoretic) treatment of the theory. From an empirical (applied) point of view, however, Cantelli agreed that the *a priori* assigning of probabilities amounts to a prediction of relative frequencies and was willing to admit a frequentist viewpoint. Hence, Cantelli did not reject the classical (Laplacian) definition of probability, but rather he inserted it into a context that was radically different from that typical of the 19th and early 20th centuries. This justifies Popper's (1983) characterization of Cantelli (together with Borel, Castelnuovo, and Kolmogorov) as a *neoclassical* author.

— Margherita Benzi, Michele Benzi and Eugene Seneta

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