1. Find the Newton form of the interpolation polynomial for the following points: \((-1, 0), (0, 1)\) and \((2, 6)\). After converting the Newton form to the (standard) power series form, sketch the graph of the resulting curve. What type of curve is it?

**Solution:**

The Newton form of the interpolation polynomial is

\[ p(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1). \]

Imposing the interpolation conditions \(p(x_i) = y_i, \ 0 \leq i \leq 2\), results in the lower triangular system

\[
\begin{bmatrix}
1 & 0 & 0 \\
1 & 1 & 0 \\
1 & 3 & 6
\end{bmatrix}
\begin{bmatrix}
b_0 \\
b_1 \\
b_2
\end{bmatrix}
= \begin{bmatrix}
0 \\
1 \\
6
\end{bmatrix}.
\]

Solution by forward substitution yields \(b_0 = 0\), \(b_1 = 1\) and \(b_2 = \frac{1}{2}\), hence the Newton interpolation polynomial is

\[ p(x) = x + 1 + \frac{1}{2}(x + 1)x, \]

or

\[ p(x) = \frac{1}{2}x^2 + \frac{3}{2}x + 1. \]

Thus, the graph of the polynomial represents a (concave-up) parabola.