1. Write down Newton’s method for finding a root of a differentiable function \( f(x) \).

**Solution:**

Newton’s method is the iteration

\[
x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, \ldots,
\]

where \( x_0 \) is an arbitrary initial guess.

2. Starting with \( x_0 = 1 \), write down the first two iterates \( x_1 \) and \( x_2 \) of Newton’s method for computing a root of \( f(x) = x^3 + x^2 - 1 \). (Note: Write down the answers as fractions. Do not use decimals.)

**Solution:**

We have \( f(x) = x^3 + x^2 - 1 \), \( f'(x) = 3x^2 + 2x \), hence Newton’s method becomes

\[
x_{n+1} = x_n - \frac{x_n^3 + x_n^2 - 1}{3x_n^2 + 2x_n}, \quad n = 0, 1, \ldots,
\]

where \( x_0 \) is an arbitrary initial guess.

With \( x_0 = 1 \) we obtain

\[
x_1 = x_0 - \frac{x_0^3 + x_0^2 - 1}{3x_0^2 + 2x_0} = 1 - \frac{1}{5} = \frac{4}{5};
\]

next, we find

\[
x_2 = x_1 - \frac{x_1^3 + x_1^2 - 1}{3x_1^2 + 2x_1} = \frac{4}{5} - \frac{19/125}{333/440} = \frac{333}{440}.
\]

Note that \( \frac{330}{440} \approx 0.756818 \), already a pretty good approximation of the true value \( x_* = 0.75487... \) of the root.