Consider the polynomial equation
\[ a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 = 0. \]

Answer the following questions:

1. True or false: Multiple roots are typically ill-conditioned.

2. True or false: Well separated roots are always well-conditioned.

For each of the two cases, provide an example to justify your answer.

Solution:

1. True. A typical example would be \( p(x) = (x - 1.5)^5. \)

2. False. An example is Wilkinson’s polynomial \( p(x) = \prod_{i=1}^{20} (x - i). \)