DETAILED SYLLABUS FOR INSTRUCTORS OF MATH 111

Text:  James Stewart,
       Single Variable Calculus, Early Transcendentals 7/e
       Effective Fall 2014
We recommend in this syllabus that you present certain examples from the text. It is more effective if you use these as guides, making small changes and not just presenting the text *verbatim*.

Keep algebra simplified—don’t multiply out unless absolutely necessary. Set a good example for your students. Resist the temptation to be too fancy or too rigorous. If you have particularly curious students, of course, you can encourage them with more sophisticated tidbits out of class. (For example, a “simple” function that is curiously discontinuous is given by \( f(x) = [x] + [-x] \).)

The Curriculum Committee believes that the following five topics are central to the course and should be covered (in depth) by everyone teaching the course.

- The conceptual and computational understanding of the limit.
- The definition of the derivative: calculating it from the definition, its meanings (slope of the tangent line, velocity, rate of change)
- Differentiation rules (including facility with the chain rule)
- Reading, setting up, and solving word problems (extrema problems and, hopefully, related rates, as well)
- Conceptual skills with graphing—understanding the first and second derivative
- Some experience with the “theoretical infrastructure” of calculus—some discussion of any of all of the following: continuity and the intermediate value theorem, maximum value theorem, mean value theorem.
- Basic antidifferentiation and integration with applications to separable volumes and the Mean Value Theorem.

**GUIDELINES FOR GRADES:**

**A:** The student has computational mastery of the course, can set up and solve non routine word problems, and has an understanding of the theoretical aspects of the course (e.g. how to apply the Intermediate Value Theorem or Mean Value Theorem).

**B:** The student has computational expertise, but may make occasional errors, can set up and solve standard word problems, and, e.g., can give an example of a non-differentiable continuous function.

**C:** The student demonstrates basic computational skills, has some conceptual understanding of the meaning of the derivative, and can do most of a routine word problem.

**D:** The student can do routine calculations, including a moderate chain rule application, can find the equation of the tangent line, but struggles to set up a routine word problem.
Outline of course

The syllabus is based on a MWF schedule. Each topic below should take a one lecture.

I. Functions and Limits (1 1/3 weeks)
   1.1 – 1.2 Introduction to Functions
   1.3 Combining Functions
   1.5 Exponential Functions
   1.6 Inverse Functions, Logarithmic and Inv. Trig. Functions

II. Limits (1 1/3 weeks)
   2.1 Tangent Lines and Velocity
   2.2 – 2.3 Limit of a Function and Limit Laws
   2.5 Continuity
   2.6 Limits at Infinity

III. Derivatives (3 weeks)
   2.7 The Derivative as a Rate of Change
   2.8 The Derivative as a Function
   —EXAM 1—
   3.1 Derivatives of Polynomials and Exponential Functions
   3.2 The Product and Quotient Rules
   3.3 Trigonometric Functions
   3.4 The Chain Rule
   3.5 Implicit Differentiation
   3.6 Logarithmic Functions
   3.7 Applications: Natural and Social Sciences

IV. Applications of Differentiation (2 weeks)
   3.8 Exponential Growth and Decay
   4.1 Global Extrema and the Extreme Value Thorem
   —EXAM 2—
   4.2 The Mean Value Theorem
   4.3 Sketching the Graph of a Function: Part 1
   4.4 Indeterminate Forms and L'Hospital's Rule
   4.5 Sketching the Graph of a Function: Part 2
   4.7 Optimization

V. Integration (2 weeks)
   4.9 Antiderivatives
5.1 Areas and Distances
5.2 Definite Integrals
5.3 Fundamental Theorem of Calculus
   —EXAM 3—
5.4 Indefinite Integrals
5.5 The Substitution Rule

VI. Applications of Integration (1 week)

6.1 Areas Between Curves
6.2 Volumes
6.5 Average Value of a Function
Outline of Exam Topics

Exam 1

(1) Functions
   (a) Composition
   (b) Expanding logarithms
   (c) Simplifying expressions such as $\sin(\arctan x)$

(2) Limits
   (a) Solve by factoring
   (b) Solve by multiplying by conjugate
   (d) Infinite limits

(3) Continuity
   (a) Finding points of continuity (usually of a piecewise defined function)
   (b) Using the Intermediate Value Theorem to prove existence of a solution of an equation

(4) Computation of derivatives from the definition for a simple rational function or a function involving square root

(5) Application of tangent line and/or rate of change

Exam 2

(1) Computation of derivatives using sum, product, quotient, generalized power rules and derivatives of trig functions

(2) Computation of derivatives using the chain rule

(3) Implicit Differentiation

(4) Logarithmic Differentiation

(5) Applications
   (a) Exponential Growth/Decay
   (b) Global Extrema

Exam 3

(1) The Mean Value Theorem

(2) Curve Sketching

(3) Indeterminate Limits and L’Hospital’s Rule

(4) Optimization Problem

(5) Antiderivatives and Integrals through FTOC.

Final Exam

The final exam should be cumulative.
Detailed Syllabus

§1.1 Functions

Purpose: To review the basic definition of functions and introduce several families of functions

Outline of lecture:

(1) Review functions, domains and the algebraic and graphical representations of functions, including the vertical line test.
(2) Introduce piecewise functions
(3) Derive the form of the difference quotient
(4) Explain what it means for a function to be increasing/decreasing on an interval.

In Class Problems: Ex.#2, 3, 11; (p.19) 47, 73
Core Problems: (p.19): 3-4, 7-10, 27-37, 39-50, 73-78;

§1.2 – 1.3 Cataloging the Functions

Purpose: To display the essential functions and learn how to combine them in order to create more varied functions

Outline of lecture:

(1) Introduce the families of functions (polynomials, rational functions, trigonometric functions, etc.)
(2) Introduce addition, subtraction, multiplication and division of functions
(3) Explain composition of functions.

In Class Problems: Ex.#6, 8; (p.42): 29
Core Problems: (p.33): 1-4; (p.42): 29-49, 51

§1.4 Exponential Functions

Purpose: To explore the definition and properties of exponential functions.

Outline of lecture:

(1) Start by defining the exponential function $a^n$, where $n$ is an integer.
(2) Work up to $a^{p/q}$, where $p,q$ are integers.
(3) Extend the definition of $a^x$ to the case when $x$ is irrational.
(4) Give the students a glimpse of motivation for limits.
(5) State the laws of exponents, and demonstrate with examples.
(6) Introduce the constant $e$ as the number for which $f(x) = e^x$ has a tangent line at $(0,1)$ with slope 1.

In Class Problems: Ex.#1; (p.57): 4
Core Problems: (p.57): 1-4, 11-16, 19-24, 29-30
§1.6 Inverse Functions

Purpose: To set up general inverse functions and define logarithmic functions, as well as inverse trigonometric functions.

Outline of lecture:

(1) Explain what it means for a function to be one-to-one.

(2) Show the horizontal line test for a function to be one-to-one.

(3) Introduce invertible functions using the notion of one-to-one and link inverse functions to the cancellation laws.

(4) Walk through the steps involved in computing an inverse function.

(5) Introduce logarithmic functions as inverse to the exponential functions.

(6) Note that by restricting the domain of a function, which is not one-to-one, it is possible for the function to become one-to-one, and use this to explain the existence of inverse trigonometric function.

In Class Problems: Ex.#1, 2, 6, 8, 12, (p.69): 22

Core Problems: (p.69): 9-12, 15-18, 21-26, 35-41, 51-56, 63-68

§2.1 The Tangent and Velocity Problems

Purpose: To motivate the exploration of limits with a secondary purpose of motivating the application of limits to define the derivative.

Outline of lecture:

(1) Discuss the tangent problem. Start with the definition of the line given two points in the plane. Then investigate what happens when you move one point closer to the second.

(2) Introduce the velocity problem. Using the position function, show the relationship between the average velocity over an interval and the slope of the line.

(3) Show that taking the average velocity over smaller and smaller intervals yields the instantaneous velocity.

(4) Emphasize at each juncture that algebra alone is inadequate to calculate the slope of the tangent line or instantaneous velocity due to its inability to deal with the concept of infinity.

In Class Problems: Ex.#1, 3

Core Problems: (p.86): 1, 2, 5, 7
§2.2 – 2.3 The Limit

*Purpose:* To formally define the limit and be able to use limit laws to calculate limits

*Outline of lecture:*

1. Concentrate Section 2.2 on setting up notation and conceptual understanding. First define the limit.
2. Emphasize that the limit does not always exist.
3. Define the one-sided limits and demonstrate that the limit exists if and only if both one sided limits exist and agree.
4. Introduce infinite limits.
5. Demonstrate the basic limit laws and show how they can be used to calculate the limit of any polynomial.
6. Show how the limit laws can sometimes be applied to rational functions, when the denominator doesn’t tend to zero.
7. When the limit of the denominator is zero, demonstrate that sometimes algebra can be used to reduce the rational function to a form where the limit laws apply, and sometimes the limit does not exist.
8. Introduce the squeeze theorem.

*In Class Problems:* §2.2: Ex.#1, 4, 6; §2.3: Ex.#2, (p.106): 11, 13, 21

*Core Problems:* (p.96): 4-9, 29-36; (p.106): 1-9, 11-32, 48-50

§2.5 Continuity

*Purpose:* To formalize the intuitive notion of continuity of a function with the formal definition using limits.

*Outline of lecture:*

1. Define continuity of a function at a point, underscoring the three requirements which must be checked.
2. Use the examples to show how discontinuities can be detected and the types of discontinuities that can occur.
3. Briefly discuss one-sided continuity, and the natural way in which the definition is altered.
4. Transition from point-wise continuity to continuity on an interval
5. State Theorems 4 – 7
6. Given a generic function, show how to use the previous point to calculate its domain of continuity.
7. Demonstrate the Intermediate Value Theorem.

*In Class Problems:* Ex.#1, 2, 10; (p.127): 20, 26, 36

*Core Problems:* (p.127): 12-14, 17-32, 41-43, 51-54
§2.6 Limits at Infinity and Horizontal Asymptotes

*Purpose:* To better understand the concept of infinity, and extend the definition of a limit to include limits at infinity

*Outline of lecture:*

1. Define the limit at infinity
2. Show an example where the limit exists. Then sketch the graph and reveal that the existence of a limit at infinity is interpreted graphically as a horizontal asymptote.
3. List some of the essential functions which have limits at one or both of the infinities (e.g., \( \arctan x \), \( e^x \), etc.)
4. Work through some examples of computing limits at infinity involving rational functions.

*In Class Problems:* Ex.#2, 3, 10; (p.140): 19, 35

*Core Problems:* (p.140): 3, 4, 15-38

§2.7 Rates of Change: Intro. to Derivatives

*Purpose:* To introduce the point-wise definition of the derivative, linking the concept of limits to the problems in Section 2.1

*Outline of lecture:*

1. Recall the slope of the secant line from Section 2.1 and the idea leading up to finding the slope of the tangent line.
2. Define the slope of the tangent line of a function at a point by applying limits.
3. Show examples and demonstrate how both forms of the difference quotient work.
4. Use the difference quotient and the limits to compute the instantaneous velocity.

*In Class Problems:* Ex.#1, 2, 3; (p.150): 5, 27

*Core Problems:* (p.150): 3, 5-8, 13, 15, 17-20, 27-32, 39, 40
§2.8 The Derivative as a Function

Purpose: To use the point-wise definition of the derivative of a function to construct a function, the derivative

Outline of lecture:

(1) Define the derivative as a function.

(2) Demonstrate that the definition is that of a function by determining the domain and sketching the graph of the derivative of an example function.

(3) Show the relationship between the domain of a function and the domain of its derivative.

(4) Show the various notations used to express the derivative, and explain what it means for a function to be differentiable.

(5) Discuss the relationship between continuity and differentiability for a function and the various ways in which a function can fail to be differentiable.

(6) Introduce the higher derivatives.

In Class Problems: Ex.#1, 4, 5, 6
Core Problems: (p.162): 3-11, 21-31, 43-48

§3.1 Derivatives of Polynomials and Exponential Functions

Purpose: To begin the investigation of systematically determining formulas for derivatives of functions

Outline of lecture:

(1) Compute, using limits, the derivatives of the constant functions, as well as $f(x) = x$.

(2) Recall from previous sections the derivatives of $x^2$, $x^3$ and manually compute the derivative of $x^4$. Note the emerging pattern, and explain that $\frac{d}{dx}x^n = nx^{n-1}$ for any value of $n$, not just positive integers.

(3) State the fact that the derivative distributes over addition and subtraction, as well as constant multiples.

(4) Use the previous facts to compute the derivatives of polynomials, and some more general sums of power functions.

(5) Compute the derivative of the exponential function using the difference quotient and our specific choice for the number $e$ that was made in Section 1.5.

In Class Problems: Ex.#2; (p.181): 8, 14, 18, 19
Core Problems: (p.181): 3-36
§3.2 The Product and Quotient Rules

**Purpose**: To derive a way to differentiate the product or quotient of two functions

**Outline of lecture**:

1. Derive the formula for the product rule, using the delta notation, and difference quotient
2. Work through some examples, demonstrating the use of the product rule.
3. State and apply the quotient rule. Caution that the order of the subtraction in the formula matters.

*In Class Problems*: Ex.#1; (p.189): 12, 13, 24, 6

*Core Problems*: (p.189): 3-32

§3.3 Derivatives of Trig. Functions

**Purpose**: To determine formulas for the derivatives trigonometric functions

**Outline of lecture**:

1. Sketch the derivative of \( \sin x \) and have the students guess at the function to which the sketch belongs.
2. Verify the prediction by computing the derivative of \( \sin x \). Note this will take a good portion of the lecture.
3. Emphasize the limit of \( \frac{\sin x}{x} \) as \( x \to 0 \) as being important on its own.
4. State the derivative of \( \cos x \) and show how to apply the quotient rule to determine the derivative of \( \tan x \).
5. State the derivatives of \( \sec x, \csc x, \tan x, \cot x \).

*In Class Problems*: Ex.#1, 6; (p.197): 8, 40

*Core Problems*: (p.197): 1-16, 21-24, 39-48

§3.4 The Chain Rule

**Purpose**: Learn how to differentiate composite functions

**Outline of lecture**:

1. Derive the chain rule using the difference quotient.
2. Work through several examples.
3. Complete the determination of the derivative of the general exponential function which was started in Section 3.1.
In Class Problems: Ex.#1, 8; (p.205): 10, 18, 44
Core Problems: (p.205): 1-54

§3.5 Implicit Differentiation

Purpose: To investigate implicitly defined functions and determine a method for computing the derivatives of such functions

Outline of lecture:

(1) Introduce implicit functions.

(2) Show by example that it is not always possible to transform implicit functions into explicit functions.

(3) State that \( \frac{dx}{dy} = \frac{dy}{dx} \) and use this fact, together with the chain rule, to explain implicit differentiation.

(4) Use implicit differentiation to compute the derivatives of the inverse trig. functions.

(5) Explain how to compute the second derivative of an implicit function.

In Class Problems: Ex.#1, 2, 4; (p.215): 11,
Core Problems: (p.215): 1-20, 25-32, 35-40, 49-60

§3.6 Derivatives of Logarithmic Functions

Purpose: To compute the derivatives of logarithmic functions, and understand how and when to use logarithmic differentiation

Outline of lecture:

(1) Apply implicit differentiation to compute the derivative of the general logarithm

(2) Note the special case of the natural logarithm.

(3) Demonstrate with an example, the motivation for logarithmic differentiation, as opposed to using a combination of quotient, power, and chain rules.

(4) Explain logarithmic differentiation.

In Class Problems: Ex.#7, 8; (p.223): 2, 4
Core Problems: (p.223): 2-34, 39-50
§3.7 Rates of Change in the Natural and Social Sciences

*Purpose:* To demonstrate several direct applications of derivatives in Physics, Chemistry, Biology and Economics

*Outline of lecture:*

1. For Physics, recall the velocity, extend this to acceleration and also discuss electrical current.
2. For Chemistry, discuss the rate of reaction for a chemical reaction, as well as compressibility.
3. For Biology, discuss the population growth rate, and, if there is time, the velocity gradient for blood flow.
4. For Economics, discuss marginal cost/revenue/profit.

*In Class Problems:* Ex.#1-8

*Core Problems:* (p.233): 1-4, 7-10, 31, 32

§3.8 Exponential Growth and Decay

*Purpose:* To compute the size of a particular quantity for which we know that the rate of growth is proportional to its current size. The direct applications presented are population growth, radioactive decay and compound interest.

*Outline of lecture:*

1. Starting with the assumption that the derivative is proportional to the size of a quantity, write down a formula for the derivative.
2. Give the general solution to this differential equation.
3. Investigate a pure growth model of population growth and the model for radioactive decay by applying the assumption and then computing the relative growth rate.
4. Start with simple interest and derive the formulas for compound interest with $n$ compounds.
5. Introduce continuous compounding by letting $n \to \infty$.
6. If time permits, explain Newton’s law of cooling.

*In Class Problems:* Ex.#2, 4; (p.242): 2

*Core Problems:* (p.242): 1-6, 8-11, 18-20
§4.1 Maximum and Minimum Values

Purpose: To use derivatives to find and classify local maxima and minima and find the global maximum and minimum over a closed interval

Outline of lecture:

(1) Define both the global and local extrema, and give graphical examples of generic functions.

(2) Observe that the local extreme values of a differentiable function seem to have horizontal tangent lines. Then show Fermat’s Theorem, stating this as a fact.

(3) Give an example which shows that the converse of Fermat’s Theorem is false (e.g. \( f(x) = x^3 \))

(4) Introduce critical points, and tie the idea of classifying critical points to the problem encountered in (3)

(5) Explain the extreme value theorem, giving the step by step procedure for determining the location of the maximum and minimum values.

In Class Problems: Ex.#2, 3, 5, 6; (p.280): 48, 61

Core Problems: (p.280): 29-44, 47-62

§4.2 The Mean Value Theorem

Purpose: To understand the mean value theorem as a general principle which drives the idea behind Fermat’s theorem

Outline of lecture:

(1) Introduce Rolle’s Theorem. It is easy to make the connection between critical points and Rolle’s Theorem.

(2) Introduce the Mean Value Theorem, and show graphically how it generalizes Rolle’s Theorem.

(3) Show example in which you locate the position which satisfies the MVT.

(4) State Theorem 5 and Corollary 7 as facts that owe their existence to the Mean Value Theorem without proof (of course), if time allows.

In Class Problems: Ex.#2, 3, 5

Core Problems: (p.288): 1-15, 17-20
§4.3 How Derivative Affect the Shape of a Curve

**Purpose**: To learn how to use the first and second derivatives to understand where a function is increasing/decreasing and its concavity

**Outline of lecture**:

1. Introduce graphically what it means for a function to be increasing or decreasing on an interval, and use this to motivate the increasing/decreasing test.

2. If there is time, you can prove this to them as a simple yet important application of the Mean Value Theorem.

3. Outline the procedure for determining the intervals of increase/decrease for a function.

4. Show that by determining the intervals of increase/decrease leads to the first derivative test for classifying critical points.

5. Introduce the idea of concavity of a function by showing graphically how a function can increase (or decrease) in two different ways, one corresponding to each type of concavity.

6. State the concavity test and show how to find the intervals of concavity for a function, introducing inflection points.

7. State the second derivative test as an alternate way to classify critical points.

**In Class Problems**: (p.297): 16 (weave into lecture), 20

**Core Problems**: (p.297): 9-21, 33-52

§4.4 Indeterminate Forms and l’Hospital’s Rule

**Purpose**: To use derivatives to evaluate limits of an indeterminate form. This finishes the discussion of limits, and it is also the last tool needed to understand the shape of the curve of a function.

**Outline of lecture**:

1. Motivate the lecture by trying to evaluate an indeterminate limit which cannot be simplified by simple algebra (e.g. \( \lim_{x \to 1} \ln x / (x - 1) \))

2. Introduce l’Hospital’s Rule as a solution to the problem.

3. Walk through each of the five other indeterminate forms of a limit. Be very careful and clear that l’Hospital’s Rule cannot be applied directly to any of these forms.

4. For each indeterminate form, show them how to manipulate the function algebraically to put it into a fractional form that l’Hospital’s Rule can deal with.

**In Class Problems**: Ex.#1; (p.307): 14, 44, 54, 56

**Core Problems**: (p.307): 7-66
§4.5 Summary of Curve Sketching

*Purpose:* To put together all of the tools from the chapter to sketch the graph of a function

*Outline of lecture:*

(1) Walk through the step-by-step outline of sketching a curve, showing each step in an example.

*In Class Problems:* Ex.#1; (p.317): 30
*Core Problems:* (p.317): 1-54

§4.7 Optimization Problems

*Purpose:* As it has been established that derivatives can be used to find maximum and minimum values, this section applies that idea to problems which are more naturally stated. The goals here are first to translate an optimization problems into finding the max/min value of a specific function, and then to apply what we have learned to solve and interpret the solution.

*Outline of lecture:*

(1) Introduce the 6 step approach to solving optimization problems.

(2) Work through several examples.

*In Class Problems:* Ex.#1-4
*Core Problems:* (p.331): 2-8, 11-17, 19-21, 32-38, 46, 48, 7-10

§4.9 Antiderivatives

*Purpose:* As a final application of differentiation and a setup for integration, the natural question of “working backwards” from a derivative is investigated

*Outline of lecture:*

(1) Introduce the concept of the antiderivative.

(2) Show that for a given function, an antiderivative is not unique.

(3) Stress the importance of the constant C, this is the moment that will define how the students view the importance of this constant for the rest of their lives, so be emphatic!

(4) Show the antiderivatives for the essential functions, as well as showing that the antiderivative distributes over addition, subtraction and constant multiples, just like the derivative.

(5) Stress that antidifferentiation does not distribute over multiplication, division or composition of functions.

(6) Introduce rectilinear motion as an application.

*In Class Problems:* Ex.#1, 2, 3; (p.348): 22, 62
*Core Problems:* (p.348): 1-48, 59-64
§5.1 Areas and Distances

Purpose: To introduce the motivation for integration: calculating areas of figures too complex for geometry alone. There is a nice trend developing. Limits helped algebra overcome an obstacle by defining a derivative, and now it will help geometry overcome its shortcomings by defining the integral.

Outline of lecture:

(1) The focus should be on defining the problem of finding areas of regions between a curve and the $x$-axis and on the Riemann sums. Don’t rush to link this to antiderivatives just yet.
(2) Show how you can compute the area of a complex polygon by dividing it into triangles and summing the results.
(3) Approach the main problem: determining the area beneath a curve, and propose an approximation by using rectangles.
(4) Show that as the number of rectangles increases (and hence the width of each decreases) the error in the approximation diminishes.
(5) Take this thought to its natural conclusion: if there were an infinite number of rectangles, the “approximation” would be perfect. Since it is not possible, limits come to the rescue!
(6) Walk through the notation surrounding the summation notation.
(7) As a secondary motivation problem, introduce the distance problem, and show that it is actually just the area problem again graphically.

In Class Problems: Ex.#2, 4
Core Problems: (p.369): 1-8, 19-23

§5.2 The Definite Integral

Purpose: Continue the exploration of the area problem, giving a name to its solution: the definite integral. Among other things, this section demonstrates how difficult calculating a definite integral directly is, setting up for the FTOC

Outline of lecture:

(1) Define what it means for a function to be integrable and the definite integral, and show the corresponding notation. DO NOT spend time on the precise definition.
(2) Be sure to explain that the area beneath a curve will be computed with a negative sign and the corresponding difference between area and net area.
(3) The amount of time spent evaluating integrals using the Riemann sums and limits will depend on the emphasis you wish to place on them for the quizzes/exams. In any case, you should show at least one example to demonstrate that simplifying the sums depends on something extremely nontrivial.
(4) Begin the process of streamlining the process of evaluating definite integrals by introducing the properties.

In Class Problems: Ex.#1, 2, 6; (p.382): 37
Core Problems: (p.382): 17-24, 33-40, 47-50, 55-58
§5.3 The Fundamental Theorem of Calculus

_Purpose_: To show that the problem of calculating the definite integral and the antiderivative are intricately linked. Therefore, to solve the integration problem, we can focus on calculating antiderivatives, which turns out to be much easier.

_Outline of lecture:_

(1) Introduce the function of _x_: \( \int_0^x f(t) dt \). Do this carefully. Since the integral notation is new, students tend to have a difficult time wrapping their heads around putting a variable in the limits.

(2) Show, through example, that the graph of this function seems to have the same features as the antiderivative of _g(t)_.

(3) Introduce the FTOC, part 1, confirming the previous observation.

(4) State the FTOC, part 2, which provides a method for computing definite integrals using tools (the antiderivative) that we have already developed.

_In Class Problems:_ Ex.#1, 6, 7; (p.394): 7, 57, 40

_Core Problems:_ (p.394): 5-44, 55-59

§5.4 Indefinite Integrals and the Net Change Theorem

_Purpose_: To introduce a new name for the antiderivative: the indefinite integral, and to examine the definite integral as the net change of a function.

_Outline of lecture:_

(1) Carefully introduce the definition and notation for the indefinite integral, pointing out the similarities and differences with that of the definite integral.

(2) Recall that by the FTOC the indefinite integral of a function is the antiderivative of the function, and bring forward all of the rules developed in §4.9 (including adding +C due to the lack of uniqueness of an antiderivative).

(3) Show how the FTOC allows us to view the definite integral as the net change of the antiderivative of a continuous function over an interval.

_In Class Problems:_ Ex.#3, 6; (p.403): 32, 41, 42

_Core Problems:_ (p.403): 1-18, 21-43 (except 40)
§5.5 The Substitution Rule

*Purpose:* To introduce a technique to handle computing the antidervative of a composition of two functions.

*Outline of lecture:*

1. Begin by presenting an indefinite integral that we cannot yet evaluate with our given toolset, and make note of the key feature: there is a composition of two functions.

2. Walk through the solution of the problem using the substitution method.

3. Show the general form for an indefinite integral for which we can apply the substitution rule, and illustrate the general solution of substituting the inner function.

4. Be sure to stress the skill of being able to dissect compositions as necessary to identifying and applying the substitution rule.

5. Demonstrate the two ways to properly handle the limits for a definite integral when applying the substitution rule.

*In Class Problems:* Ex.#1, 6; (p.413): 13, 44, 54, 60

*Core Problems:* (p.413): 1-48, 53-63

§6.1 Areas Between Curves

*Purpose:* To show how the idea of finding the area between a single curve and an axis can be generalized to finding the area between two curves.

*Outline of lecture:*

1. Start by examining the problem through the lens of approximation with rectangles and Riemann sums.

2. Conclude that the exact area is given by an integral, and state it.

3. The rest of the lecture is example driven. Be sure to demonstrate:
   - How to handle intersections in the curves, and what can happen if you are not careful, and
   - that two functions of $y$ work just as easily as two functions of $x$.

*In Class Problems:* Ex.#1, 2, 5; (p.427): 12

*Core Problems:* (p.427): 1-30
§6.2 Volumes

*Purpose:* To demonstrate that the process of approximating a computation with a Riemann sum and then translating to an integral extends beyond areas. In particular, we see how the problem of calculating volumes can be solved in this manner.

*Outline of lecture:*

1. Recall the volumes of some basic geometric shapes (cylinders will be necessary for the approximation of the general solid of revolution).
2. Show how to approximate the volume of a general solid by dividing it into slices, and use this to build a Riemann sum to define the exact volume.
3. Introduce the solids obtained by rotating a curve around an axis, and note that the shape of the slices in this case is a cylinder, making the integral “easy” to solve.
4. Show that if the region rotated around the axis is between two curves, the slices have the shape of “washers,” that is a cylinder with a smaller concentric circle removed.
5. If there is time, you may wish to show that you can apply the Riemann sum technique to any solid which has slices, whose general area is simple to compute (see Ex. 8)

*In Class Problems:* Ex.#2, 4; (p.439): 5

*Core Problems:* (p.438): 1-18

§6.5 Average Value of a Function

*Purpose:* To demonstrate a non-geometric problem which can also be approached with the concept of integration, computing the average value of a function over an interval.

*Outline of lecture:*

1. Introduce the average value, first recalling the more familiar average of a finite set of values, and then extending to the average value of a function over an interval. Demonstrate the latter graphically.
2. Show how Riemann sums can once again be used to bridge the gap between an average of a finite set, and the average of a function.
3. Define the average value of a function as an integral.
4. Introduce the Mean Value Theorem.

*In Class Problems:* Ex.#1, 2

*Core Problems:* (p.453): 1-12