

**Math 211 “Multivariable Calculus with Physics Applications”**  
**Emory University**  
**Fall 2009**

**Instructor:** Brussel (MSC W414; 7-5605)

**Text:** “Multivariable Calculus”, by McCallum, Hughes-Hallet, Gleason, et al. (5th Ed.)

**Class:** MSC W303, TTh 10:00-11:15.

**Introduction:** Functions are mathematical representations of real world processes. The input is the “cause” and the output is the “effect”. In one-variable calculus, the function is  $f$ , the input is  $x$ , and the output is  $y = f(x)$ . We write  $f : \mathbb{R} \rightarrow \mathbb{R}$ , where the first  $\mathbb{R}$  is the  $x$ -axis, and the second  $\mathbb{R}$  is the  $y$ -axis. One dimensional input, one dimensional output. The limitation of this viewpoint is that it only treats *primitive phenomena*. We want to model phenomena that have *multiple* inputs and outputs. For example, we want to model phenomena whose input includes their location in space,  $\mathbb{R}^3$ . Since a point in space is given by three numbers  $x, y, z$ , that’s a three-variable input, leading us to study three-variable functions  $f(x, y, z)$ . If the output is a single number, we’ll write  $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ . This could model, for example, temperature in the earth’s atmosphere. If the output is also three-dimensional, we write  $f = (f_1(x, y, z), f_2(x, y, z), f_3(x, y, z))$ , and  $f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ . This could model fluid flow in an artery, or the gravitational field in space.

Our goal is *Stokes’ Theorem*, a beautiful and powerful generalization of *The Fundamental Theorem of Calculus*. To reach it, we’ll study the geometry of surfaces in space, introduce vector fields, and learn how to differentiate and integrate over curves, surfaces, and solids. In the end, we’ll use our elegant theory to solve interesting problems in classical physics. Yet *no physics background is required*.

To take this course you must have a very good grip on one-variable calculus (a list of assumed skills is on the next page). Our approach will be geometrical, and it will involve a more sophisticated type of thinking than you may have seen in math courses. Less symbolic manipulation, more geometric visualization. Be prepared for a sudden increase in depth.

**Requirements:** You must attend all lectures, complete all problems in the weekly homework assignments, and take a quiz, two midterms, and a final. Assume the final grade will be determined by the formula  $\text{Total Score} = .25 \cdot \text{HW} + .05 \cdot \text{Q} + .2 \cdot (\text{MT 1} + \text{MT 2}) + .3 \cdot \text{F}$ .

**Homework:** To do well, you must do *all* of the homework problems, and show all of your work. All. If you get stuck, you *must* get help. Push yourself to think about the problems *geometrically* as well as computationally. Development of geometric intuition is crucial in this course.

Homework is assigned every Thursday, and due the following Thursday, in class. Late homework will be acknowledged but not graded. You won’t get credit if you don’t show your work. You may work with others on homework problems, but *you must write up your own original set*. Copying will anger the instructor, and is considered by him to be a violation of the Emory College Honor Code.

**Office Hours:** A good opportunity to talk to me and to others in the class about the homework problems. T, W 8:30-9:45am.

**Test Dates:** Quiz: Thursday September 17  
Midterm 1: Thursday October 1  
Midterm 2: Thursday November 5  
Final Exam: Tuesday December 15, 4:30-7:00

*If you can’t make any of these dates, you must notify me in advance.*

## General Prerequisites for Math 211

Every student of Math 211 should be familiar with the topics on the following list. If you don't remember some of them, you should review; they will definitely come up.

1. Coordinate systems: Euclidean  $\mathbb{R}^3$ ,  $\mathbb{R}^2$ ; polar  $\mathbb{R}^2$ .
2. Trigonometry: Pythagorean Theorem; interpretation of  $\sin \theta$ ,  $\cos \theta$  on unit circle; double angle formula.
3. Area formulas for circle, cylinder, sphere; volume formulas for rectangular box, cylinder, sphere.
4. Vectors: Addition (geometric, and by components);  $\vec{i}, \vec{j}, \vec{k}$ ; length formula.
5. Matrices ( $2 \times 2$ ) and their action on vectors.
6. Solution of system of (two) linear equations.
7. Physics: speed·time=distance; force·distance=work.
8. Fundamental Theorem of Calculus  $\int_a^b \frac{df}{dx} dx = f(b) - f(a)$ .
9. Integration:  $u$ -substitution;  $\int \sin^2(\theta) d\theta$ ; how to find the area between two graphs; integrals of  $x^n$  ( $n \in \mathbb{Z}$ ),  $\sin x$ ,  $\cos x$ ,  $e^x$ .
10. Differentiation: Chain rule; product rule; power rule; derivatives of  $\sin x$ ,  $\cos x$ ,  $e^x$ ,  $\ln x$ .
11. Riemann sums; how to estimate a definite integral.
12. The limit of a simple ratio of polynomials.
13. How to find and classify critical points of a function in 1-variable.
14. The equation of a tangent line.
15. Geometry: equations and graphs of circle, ellipse, hyperbola, parabola; graphs of  $e^x$ ,  $\ln x$ ,  $\sin x$ ,  $\cos x$ ,  $\tan x$ ,  $x^2$ ,  $1/x$ ; graphs in polar coordinates, like  $r^2 = 4 \cos 2\theta$ .

## Rough Syllabus for 26 Classes

1. Introduction, vectors, coordinates, dot product/projection, matrices, determinants and volume distortion, orthogonal transformations, cross product.
2. Vector equations for lines and planes, parameterized curves, velocity/acceleration, arc-length.
3. Vector fields, line integrals, computing work on piecewise smooth paths through nice force fields.
4. Surfaces, parameterized surfaces, cross sections, quadric surfaces.
5. Partial derivatives, tangent plane, directional derivatives and the gradient, examples.
6. Chain rule, double integrals (order of integration, volume, average value).
7. Examples, Jacobians, double integrals in polar coordinates. Triple integrals in rectangular coordinates (volume, mass, average value).
8. Cylindrical and spherical coordinate systems, more Jacobians, triple integration in cylindrical and spherical coordinates, examples.
9. Circulation, flux, curl and divergence of a vector field in  $\mathbb{R}^2$ , Green's theorem for flux *and* circulation. Curl/circulation computations in  $\mathbb{R}^2$  for three circle-symmetric fields around circles, also of  $\vec{f} = \frac{1}{r^2}(-y\vec{i} + x\vec{j})$  along *any* path. Flux/divergence computations in  $\mathbb{R}^2$  for three radial fields.
10. Conservative fields and path independence (5 equivalent conditions). Integration on surfaces, parameterized surfaces and area distortion, examples.
11. Flux/divergence computations in  $\mathbb{R}^3$  for radially symmetric fields, qualitative analysis.
12. Flux out of surfaces, divergence theorem, examples using radially symmetric fields and spheres, Gauss' law ("static" heat flow and drum membrane applications), point source fields.
13. Circulation on a surface, Stokes' theorem, examples, including "broken bottle".