CS171 Introduction to Computer Science II

Hash Tables
Review

- Sequential search using linked list
- Binary search using ordered arrays
- Binary search tree (BST)
- 2-3 tree (balanced search tree, implemented as red black tree)
## ST implementations: summary

<table>
<thead>
<tr>
<th>implementation</th>
<th>guarantee</th>
<th></th>
<th></th>
<th>ordered iteration?</th>
<th>operations on keys</th>
</tr>
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<tbody>
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Hashing: basic plan

Save items in a **key-indexed table** (index is a function of the key).

**Hash function.** Method for computing array index from key.

\[ \text{hash("it")} = 3 \]

**Issues.**

- Computing the hash function.
- Equality test: Method for checking whether two keys are equal.
Hashing: basic plan

Save items in a key-indexed table (index is a function of the key).

Hash function. Method for computing array index from key.

```
hash("it") = 3
```

Issues.

- Computing the hash function.
- Equality test: Method for checking whether two keys are equal.
- Collision resolution: Algorithm and data structure to handle two keys that hash to the same array index.

Classic space-time tradeoff.

- No space limitation: trivial hash function with key as index.
- No time limitation: trivial collision resolution with sequential search.
- Space and time limitations: hashing (the real world).
Hash Tables

- A **hash table** for a given key type consists of
  - Hash function \( h \)
  - Array (called table) of size \( N \)
Computing the hash function

**Idealistic goal.** Scramble the keys uniformly to produce a table index.
- Efficiently computable.
- Each table index equally likely for each key.

![Diagram](image)

**Ex 1.** Phone numbers.
- Bad: first three digits.
- Better: last three digits.

**Ex 2.** Social Security numbers.
- Bad: first three digits.
- Better: last three digits.

**Practical challenge.** Need different approach for each key type.
Example

• We design a hash table for storing entries as (SSN, Name), where SSN (social security number) is a nine-digit positive integer.

• Our hash table uses an array of size $N = 10,000$ and the hash function $h(x) = \text{last four digits of } x$. 

\begin{tabular}{|c|c|}
\hline
Index & Value \\
\hline
0 & \emptyset \\
1 & 025-612-0001 \\
2 & 981-101-0002 \\
3 & \emptyset \\
4 & 451-229-0004 \\
\cdots & \\
9997 & \emptyset \\
9998 & 200-751-9998 \\
9999 & \emptyset \\
\hline
\end{tabular}
Hash Functions

• A hash function is usually specified as the composition of two functions:

  **Hash code:**
  \[ h_1: \text{keys} \rightarrow \text{integers} \]

  **Compression function:**
  \[ h_2: \text{integers} \rightarrow [0, N - 1] \]

• The hash code is applied first, and the compression function is applied next on the result, i.e.,
  \[ h(x) = h_2(h_1(x)) \]
Java’s hash code conventions

All Java classes inherit a method \texttt{hashCode()}, which returns a 32-bit int.

Requirement. If \texttt{\textbf{x.equals(y)}}\texttt{\textbf{,}} then \texttt{(x.hashCode() == y.hashCode())}.

Highly desirable. If \texttt{!\textbf{x.equals(y)}}\texttt{\textbf{,}} then \texttt{(x.hashCode() != y.hashCode())}.

Default implementation. Memory address of \texttt{x}.

Trivial (but poor) implementation. Always return 17.

Customized implementations. Integer, Double, String, File, URL, Date, ...

User-defined types. Users are on their own.
Implementing hash code: strings

```java
public final class String {
    private final char[] s;
    ...

    public int hashCode() {
        int hash = 0;
        for (int i = 0; i < length(); i++)
            hash = s[i] + (31 * hash);
        return hash;
    }
}
```

- Horner's method to hash string of length $L$: $L$ multiplies/adds.
- Equivalent to $h = 31^{L-1} \cdot s^0 + \ldots + 31^2 \cdot s^{L-3} + 31^1 \cdot s^{L-2} + 31^0 \cdot s^{L-1}$.

**Ex.**

```java
String s = "call";
int code = s.hashCode();
```

$3045982 = 99 \cdot 31^3 + 97 \cdot 31^2 + 108 \cdot 31^1 + 108 \cdot 31^0$

$= 108 + 31 \cdot (108 + 31 \cdot (97 + 31 \cdot (99)))$
Implementing hash code: user-defined types

public final class Transaction implements Comparable<Transaction>
{
    private final String who;
    private final Date when;
    private final double amount;

    public Transaction(String who, Date when, double amount)
    { /* as before */ }

    ... 

    public boolean equals(Object y)
    { /* as before */ }

    public int hashCode()
    {
        int hash = 17;
        hash = 31*hash + who.hashCode();
        hash = 31*hash + when.hashCode();
        hash = 31*hash + ((Double) amount).hashCode();
        return hash;
    }
}
Hash code design

"Standard" recipe for user-defined types.
• Combine each significant field using the $31x + y$ rule.
• If field is a primitive type, use wrapper type `hashCode()`.
• If field is an array, apply to each element.  
  or use `Arrays.deepHashCode()`  
  applies rule recursively
• If field is a reference type, use `hashCode()`.

In practice. Recipe works reasonably well; used in Java libraries.
In theory. Need a theorem for each type to ensure reliability.

Basic rule. Need to use the whole key to compute hash code;  
consult an expert for state-of-the-art hash codes.
Modular hashing

Hash code. An int between $-2^{31}$ and $2^{31}-1$.

Hash function. An int between 0 and $M-1$ (for use as array index).

```java
private int hash(Key key)
{   return key.hashCode() % M;
}
```

bug

```java
private int hash(Key key)
{   return Math.abs(key.hashCode()) % M;
}
```

1-in-a-billion bug

hashCode() of "polygenelubricants" is $-2^{31}$

```java
private int hash(Key key)
{   return (key.hashCode() & 0x7fffffff) % M;
}
```
correct
Uniform hashing assumption

Uniform hashing assumption. Each key is equally likely to hash to an integer between 0 and $M - 1$.

Bins and balls. Throw balls uniformly at random into $M$ bins.

Hash value frequencies for words in Tale of Two Cities ($M = 97$)

Java’s `String` data uniformly distribute the keys of Tale of Two Cities
Collision Handling

- Separate Chaining
- Linear Probing
Separate chaining ST

Use an array of $M < N$ linked lists. [H. P. Luhn, IBM 1953]

- **Hash:** map key to integer $i$ between 0 and $M - 1$.
- **Insert:** put at front of $i^{th}$ chain (if not already there).
- **Search:** only need to search $i^{th}$ chain.

![Hashing with separate chaining for standard indexing client](image-url)
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<td>(linked list)</td>
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<td>$\lg N^*$</td>
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<td>3-5 *</td>
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</table>

* under uniform hashing assumption
Collision resolution: open addressing

Open addressing. [Amdahl-Boehme-Rocherster-Samuel, IBM 1953]
When a new key collides, find next empty slot, and put it there.

linear probing (M = 30001, N = 15000)
Linear probing

Use an array of size $M > N$.

- **Hash**: map key to integer $i$ between 0 and $M - 1$.
- **Insert**: put at table index $i$ if free; if not try $i + 1$, $i + 2$, etc.
- **Search**: search table index $i$; if occupied but no match, try $i + 1$, $i + 2$, etc.

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</table>

**Insert I**

hash(I) = 11

**Insert N**

hash(N) = 8
### Linear probing: trace of standard indexing client

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<tr>
<th>key</th>
<th>hash</th>
<th>value</th>
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**Notes:**
- Entries in red are new.
- Entries in gray are untouched.
- Keys in black are probes.
- Probe sequence wraps to 0.
- keys[] vals[]
public class LinearProbingHashST<Key, Value> {

    private int M = 30001;
    private Value[] vals = (Value[]) new Object[M];
    private Key[] keys = (Key[]) new Object[M];

    private int hash(Key key) { /* as before */ }

    public void put(Key key, Value val) {
        int i;
        for (i = hash(key); keys[i] != null; i = (i+1) % M)
            if (keys[i].equals(key))
                break;
        keys[i] = key;
        vals[i] = val;
    }

    public Value get(Key key) {
        for (int i = hash(key); keys[i] != null; i = (i+1) % M)
            if (key.equals(keys[i]))
                return vals[i];
        return null;
    }
}
Separate chaining vs. linear probing

Separate chaining.
• Easier to implement delete.
• Performance degrades gracefully.
• Clustering less sensitive to poorly-designed hash function.

Linear probing.
• Less wasted space.
• Better cache performance.
### ST Implementations: Summary

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* under uniform hashing assumption
Hashing vs. balanced search trees

Hashing.
• Simpler to code.
• No effective alternative for unordered keys.
• Faster for simple keys (a few arithmetic ops versus $\log N$ compares).
• Better system support in Java for strings (e.g., cached hash code).

Balanced search trees.
• Stronger performance guarantee.
• Support for ordered ST operations.
• Easier to implement `compareTo()` correctly than `equals()` and `hashCode()`.

Java system includes both.
• Red-black trees: `java.util.TreeMap`, `java.util.TreeSet`.
• Hashing: `java.util.HashMap`, `java.util.IdentityHashMap`.