Define a function $GR_C()$ as follows:

$$
GR_C(p_f^0) = \emptyset \quad \text{(Initial Value)}
$$

$$
GR_C(p_f^j) = \max \left\{ A(p_f^j), GR_C(p_f^{j-1}) \right\} + \frac{l_f^j}{r_f^j}
$$

$A(p_f^j) = \text{arrival time of packet } p_f^j$

$l_f^j = \text{length (bytes) of packet } p_f^j$

$r_f^j = \text{reserved output data rate for flow } f$

---

**Magic:** A scheduling algorithm belongs to $GR$ (Guarantee Rate) if the scheduling alg. guarantees that packet $p_f^j$ will be transmitted (completely) by at time $t$ for some positive constant $\beta$ for all packet $p_f^j$.

$$
\exists t \leq GR_C(p_f^j) + \beta
$$
Claim: \( FFS \) is a GPR scheduler.

(a) Let \( L_{GPR}(P_f^j) \) = completion time of packet \( P_f^j \) under GPR.

(b) \( L_{GPR}(P_f^0) = \emptyset \).

(c) Let each flow reserve a rate \( r_1, r_2, \ldots, r_m \) at the total capacity \( C \).

Then flow \( f \) receives a service rate of

\[
\frac{r_f}{r_1 + r_2 + \ldots + r_m} \cdot C
\]

\[
= \frac{C}{r_1 + r_2 + \ldots + r_m} \cdot r_f 
\leq C 
\geq r_f.
\]

(b) The packet \( P_f^j \) of length \( L_f^j \) will complete transmission in \( \leq \frac{L_f^j}{r_f} \) seconds.
(c) Question: when will $P_{f^j}$ finish if $P_{f^{j-1}}$ finish at $L_{GPS}(P_{f^j})$?

\[ L_{GPS}(P_{f^j}) - A(P_{f^j}) \leq \frac{l_f^j}{v_f} \]

(2)

Conclusion:

\[ L_{GPS}(P_{f^j}) \leq \max(A(P_{f^j}), L_{GPS}(P_{f^{j-1}})) + \frac{l_f^j}{v_f} \]

\[ \Rightarrow \beta = \phi \quad \text{(perfect fairness)} \]
May, Pavia, Stu

\[
\frac{\sum_{i=1}^{2} \left( \frac{1}{m_i} + c_1 \right)}{r_i} + \frac{f_i}{G_f + (K-1)M}
\]

\[
d_f = \text{delay of packet } f
\]

\[
d_f = \text{end to end delay}
\]

\[
G_f = \text{Guard with packet } f
\]

\[
G_f = \text{Guard with packet } f
\]

\[
(\text{CF,} \ f) \ \Rightarrow \ \text{buffer flush}
\]

Preemption delay

Theorem: