Loading of containers

(i) The system is stable if (and only if)
\[ \lambda \cdot (E(B_1) + E(B_2)) < 1. \]

(ii) Let \( \rho_i = \lambda E(B_i) \) and \( E(R_i) = E(B_i^2)/2E(B_i) \) for \( i = 1, 2 \). Then the mean waiting time is given by
\[ E(W) = \rho_1(E(R_1) + E(B_2)) + \rho_2 E(R_2) + E(L^q)(E(B_1) + E(B_2)), \]
and by Little's law we have
\[ E(L^q) = \lambda E(W). \]

Hence we get
\[ E(W) = \frac{\rho_1(E(R_1) + E(B_2)) + \rho_2 E(R_2)}{1 - \rho_1 - \rho_2}. \]

Since \( \lambda = 1/15 \) slaves per minute, \( E(B_1) = 3, \sigma(B_1) = 2, \) \( E(B_2) = 10 \) and \( \sigma(B_2) = 8 \) (so \( E(R_1) = 13/6 \) and \( E(R_2) = 41/5 \)), we find
\[ E(W) = \frac{237}{4} = 59 \frac{1}{4} \text{ min}. \]

The mean sojourn time follows by adding the mean fetch time, so
\[ E(S) = E(W) + E(B_1) = 62 \frac{1}{4} \text{ min}. \]

(iii) The system is stable if (and only if)
\[ \lambda \cdot (E(C_1) + E(C_2)) < 1. \]

(iv) Now let \( \rho_i = \lambda E(C_i) \) and \( E(R_i) = E(C_i^2)/2E(C_i) \) for \( i = 1, 2 \). For the mean sojourn time we have
\[ E(S) = (E(L) + 1)(E(C_1) + E(C_2)) + (1 - \rho_1 - \rho_2)(E(C_2) - (E(C_1) + E(C_2))) + \rho_1(E(R_1) + E(C_2) - (E(C_1) + E(C_2))) + \rho_2(E(R_2) - (E(C_1) + E(C_2))). \]

The first term at the right-hand side states that the mean (residual) service time of all slaves in the queue is \( E(C_1) + E(C_2) \). This is obviously not correct; the other three terms are corrections. For example, if on arrival the crane is idle (with
probability $1 - \rho_1 - \rho_2$) then the service time of the arriving slave is $E(C_2)$ instead of $E(C_1) + E(C_2)$. Together with Little’s law, i.e. $E(L) = \lambda E(S)$, we find

$$E(S) = \frac{\rho_1(E(R_1) + E(C_2)) + \rho_2E(R_2)}{1 - \rho_1 - \rho_2} + E(C_2) = 62\frac{1}{4} \text{ min.}$$

If we define the waiting time as the time elapsing from the arrival of a slave until the crane starts loading the container on that slave, then $S = W + C_2$ and thus

$$E(W) = E(S) - E(C_2) = 59\frac{1}{4} \text{ min.}$$

Hence, we obtain the same results as in (ii). This is no surprise by observing that the sojourn time remains the same when we act as if the service time of a slave is $C_2$ followed by $C_1$ (instead of the other way around), and if the slave may leave as soon as $C_2$ has been finished.

**Points**

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