Poisson Arrivals See Time Averages

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Let $N(t)$ be the number of customers in the queue at time $t$. The probability that the queue carries $k$ customer(s) is expressed as $\Pr\{N(t) = k\}$.

Let $P_k$ be the steady state probability for the queue having $k$ customer(s), hence, $P_k = \lim_{t \to \infty} \Pr\{N(t) = k\}$.

Let $A_k$ be the probability that the process monitor sees $k$ customer(s) in the queue just before an arrival, hence

\[
A_k = \lim_{t \to \infty} \Pr\{N(t) = k \mid \text{an arrival appears at } t^+\}
\]

\[
= \lim_{t \to \infty} \frac{\Pr\{N(t) = k, \text{ an arrival appears at } t^+\}}{\Pr\{\text{an arrival appears at } t^+\}}
\]

\[
= \lim_{t \to \infty} \lim_{\Delta t \to 0} \frac{\Pr\{N(t) = k, \text{ an arrival appears in } (t, t + \Delta t)\}}{\Pr\{\text{an arrival appears in } (t, t + \Delta t)\}}
\]

\[
= \lim_{t \to \infty} \lim_{\Delta t \to 0} \frac{\Pr\{N(t) = k\} \Pr\{\text{an arrival appears in } (t, t + \Delta t)\}}{\Pr\{\text{an arrival appears in } (t, t + \Delta t)\}}
\]

\[
= \lim_{t \to \infty} \lim_{\Delta t \to 0} \frac{\Pr\{N(t) = k\} \Pr\{\text{an arrival appears in } (t, t + \Delta t)\}}{\Pr\{\text{an arrival appears in } (t, t + \Delta t)\}}
\]

\[
= \lim_{t \to \infty} \Pr\{N(t) = k\}
\]

\[
= \lim_{t \to \infty} \Pr\{N(t) = k\}
\]

\[
= P_k
\]

$\therefore$ Poisson Arrivals See Time Averages