This exam is no book, no gadgets. You are allowed one sheet of notes (letter size, two sides). There are 40 questions, worth 90 total points. Marks will be curved so the median grade is at least a B.

Name (Print):

Signature:

This exam is my own work. I understand it is governed by the Emory Honor Code.

Choose One: for each language, indicate the first class containing it, or “none”. Here REG is the class of regular languages, CFL is the class of context free languages, DEC is the class of decidable languages, REC is the class of recognizable languages, and co-REC is the class of co-recognizable languages.

(2 pts) 1. The language \{a^i b^j : i = j\} is in:
   A. REG B. CFL C. P D. none

(2 pts) 2. The language \{a^i b^j : i + j \text{ is even}\} is in:
   A. REG B. CFL C. P D. none

(2 pts) 3. The language \{a^i b^j c^k : i = j \text{ or } j = k\} is in:
   A. REG B. CFL C. P D. none

(2 pts) 4. The language \{a^n : n \text{ is a square}\} is in:
   A. REG B. CFL C. P D. none

(2 pts) 5. The language \{a^i b^j c^k : i = j \text{ and } j = k\} is in:
   A. REG B. CFL C. P D. none

(2 pts) 6. The language \mathcal{A}_{PDA} = \{\langle P, w \rangle : P \text{ is a PDA and } P \text{ accepts } w\} is in:
   A. DEC B. REC C. co-REC D. none

(2 pts) 7. The language \mathcal{A}_{LBA} = \{\langle M, w \rangle : M \text{ is an LBA and } M \text{ accepts } w\} is in:
   A. DEC B. REC C. co-REC D. none

   Reminder: LBA is “linear bounded automaton”, a restricted kind of Turing machine (TM).

(2 pts) 8. The language \mathcal{A}_{TM} = \{\langle M, w \rangle : M \text{ is a TM and } M \text{ accepts } w\} is in:
   A. DEC B. REC C. co-REC D. none

(2 pts) 9. The language \mathcal{A}_{CFG} = \{\langle G \rangle : G \text{ is an CFG and } L(G) = \Sigma^*\} is in:
   A. DEC B. REC C. co-REC D. none

(2 pts) 10. The language \mathcal{E}_{CFG} = \{\langle G \rangle : G \text{ is an CFG and } L(G) \text{ is empty}\} is in:
    A. DEC B. REC C. co-REC D. none

(2 pts) 11. The language \mathcal{E}_{TM} = \{\langle M \rangle : M \text{ is an TM and } L(M) \text{ is empty}\} is in:
     A. DEC B. REC C. co-REC D. none

(2 pts) 12. The language \mathcal{E}_{TM} = \{\langle M, M_2 \rangle : M_1 \text{ and } M_2 \text{ are TM’s and } L(M_1) = L(M_2)\} is in:
     A. DEC B. REC C. co-REC D. none

Choose Several: for these three, circle ALL answers that apply.

(5 pts) 13. Which of these model conversions can grow exponentially larger?
    A. DFA to NFA    B. NFA to DFA    C. NFA to regexp    D. regexp to NFA    E. PDA to CFG

(5 pts) 14. Which classes are closed under complementation?
    A. REG    B. CFL    C. P    D. DEC    E. REC

(5 pts) 15. Which languages do we know are NP-hard?
    A. 2SAT    B. SUBSETSUM    C. \mathcal{A}_{TM}    D. CLIQUE_k    E. SAT

Note: for \( k \geq 1 \), CLIQUE_k means \{\langle G \rangle : G \text{ is a graph with a clique of size at least } k\}.
Fill in the Blank: fill each bank with an appropriate phrase or formula.

(2 pts) 16. Suppose \( M = (Q, \Sigma, \delta, q_0, F) \) is a DFA. Then \( \delta \) is a function from what domain (a set) to \( Q \)?

16. _______________________

(2 pts) 17. The three “regular operations” (on languages) are union, star, and what?

17. _______________________

(2 pts) 18. Suppose \( M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1) \) and \( M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2) \) are two DFA’s. Their product DFA has what set of states?

18. _______________________

(2 pts) 19. Suppose \( M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1) \) and \( M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2) \) are two DFA’s, and we design their product DFA to accept \( L(M_1) \cup L(M_2) \). What is its set of accepting (final) states?

19. _______________________

(2 pts) 20. Suppose \( N = (Q, \Sigma, \delta, q_0, F) \) is an NFA, and we convert it to an equivalent DFA \( M \), by the book’s construction. What is the set of states of \( M \)?

20. _______________________

(2 pts) 21. When defining regular expressions, the three base cases are: \( a \) (for some \( a \in \Sigma \)), \( \varepsilon \), and what?

21. _______________________

(2 pts) 22. In a generalized NFA (GNFA), each arrow is labeled by a what?

22. _______________________

(2 pts) 23. Suppose \( M = (Q, \Sigma, \delta, q_0, F) \) is a DFA. In order to win the regular pumping game for \( L(M) \), what value should \( R \) (the first player) pick as \( p \)?

23. _______________________

(2 pts) 24. Consider the regular pumping game for \( L = \{a^i b^j : i > j \} \). Suppose \( R \) picks \( p = 2 \), \( N \) picks \( w = aab \), and \( R \) picks \( x = a, y = a, z = b \). What should \( N \) finally pick, to win the game?

24. _______________________

(2 pts) 25. Let \( L \) be the language of the regular expression \( a^* b^* a^* \). What is a distinguishing suffix \( z \) for strings \( x = a \) and \( y = ab \)?

25. _______________________

(3 pts) 26. Write down a CFG for the language of all palindromes in \( \{a, b\}^* \).
   (A palindrome is a string that equals its reverse.)

26. _______________________

(2 pts) 27. Given a CFG \( G = (V, \Sigma, R, S) \), its language \( L(G) \) is the set of all terminal strings \( w \in \Sigma^* \), such that what?

27. ______________________
28. (2 pts) A CFG is **ambiguous** if some terminal string has two or more what?

29. (2 pts) A CFG is in **Chomsky normal form** if every rule is of the form $S \rightarrow \varepsilon, A \rightarrow a$, or what?

30. (2 pts) A PDA is a 6-tuple $(Q, \Sigma, \Gamma, \delta, q_0, F)$, where $\delta$ is a function from $Q \times \Sigma \times \Gamma_\varepsilon$ to what (a set)?

31. (2 pts) We used diagonalization to show what language is not decidable?

32. (2 pts) Suppose $A, B \subseteq \Sigma^*$. A mapping reduction “$A \leq_m B$” means there is a computable function $f : \Sigma^* \rightarrow \Sigma^*$, such that for all strings $w \in \Sigma^*$, we have what condition (an iff)?

33. (2 pts) Suppose $B$ is recognizable, and $A \leq_m \overline{B}$ (the complement of $B$). What does this tell us about $A$?

34. (2 pts) Suppose $f$ is a computable function from binary strings to integers, and for all $x \in \{0, 1\}^*$, $f(x) \leq K(x)$ (the Kolmogorov complexity of $x$). What can we conclude about $f$?

35. (2 pts) What is the definition of the class NP, as a union?

36. (2 pts) “$A \leq_p B$” is just like “$A \leq_m B$”, but with what additional condition on $f$?

37. (2 pts) A **verifier** for a language $A$ is a deterministic algorithm $V$, so that $A$ is the set of all strings $w$, such that $V$ does what?

38. (2 pts) NP can also be defined as the class of languages with verifiers of a particular kind. What kind of verifiers?

39. (2 pts) Suppose we want to show $L$ is NP-hard, and we know $L'$ is NP-hard. What reduction should we show?

40. (2 pts) Suppose we already know $L$ is NP-hard. What else do we need to show, in order to conclude that $L$ is NP-complete?