Errata for Introduction to Partial Differential Equations

David Borthwick
May 24, 2020

Please email any additional corrections to davidb@mathcs.emory.edu.

Chapter 1

p. 4: Equations (1.8) and (1.9)
There is a typo in the notation for the second partials. In both equations this term should be
\[ - \sum_{i,j=1}^{n} a_{ij}(x) \frac{\partial^2 u}{\partial x_i \partial x_j}. \]

Chapter 4

p. 49: Equations after (4.13)
The use of \( \partial g/\partial x \) is potentially confusing, since \( g \) depends on a single variable. This should be replaced by \( g' \) in the two unnumbered formulas.

p. 62: First equation
The factor of \( 1/4\pi \) should either appear on both sides or neither.

Chapter 5

p. 93: Problem 5.2
Frequencies are related to eigenvalues by \( \omega = \sqrt{\lambda} \), if physical constants are omitted. The problem include a reference here to §5.2.

p. 93: Problem 5.4
Repeated “that.”

Chapter 6

p. 101: After equation (6.11)
The rescaling should read \( (t,x) \mapsto (\lambda^2 t, \lambda x) \).

p. 102: Equation (6.12)
The \( x = 0 \) case should be \( C_2 \) instead of 0.

p. 103: First equation
Misplaced prime on the right. The first line should read:
\[ \int_{-\infty}^{\infty} \varphi'(z) \Theta(x - z) \, dz = \int_{-\infty}^{x} \varphi'(z) \, dx \]
p. 105: First equation
Missing factor of $1/4$ in the exponent. The first line should read:

$$u(t, x) = (4\pi)^{-\frac{n}{2}} \int_{\mathbb{R}^n} e^{-|w|^2/4} g(x + \frac{1}{2} tw) d^n w.$$  

p. 108: Equation (6.28)
The second term should have $f$ instead of $\partial f / \partial t$, so that the formula reads

$$\frac{\partial u}{\partial t}(t, x) = \int_0^t \int_{\mathbb{R}^n} H_s(y) \frac{\partial f}{\partial t}(t - s, x - y) d^n y \, ds$$
$$+ \int_{\mathbb{R}^n} H_t(y) f(0, x - y) d^n y,$$

p. 108: Equation (6.30)
It should be pointed out that this holds independently of $\varepsilon$, for $\varepsilon > 0$.

p. 109: Exercise 6.2
The boundary condition is incompatible with the initial condition. It should be

$$u(t, 0) = A \sin(\omega t).$$

p. 136: Exercise 8.2
In the formula for $c_k[h]$, the factor of $(-1)^k$ is irrelevant because $\sin(\pi k/2) = 0$ if $k$ is odd.

p. 144: Corollary 8.7
The norm on the right-hand side of the identity should be squared.

p. 147: Equation (8.42)
In (8.42) and the unnumbered equation preceding it, $t$ should be replaced by $y$.

p. 158: Sentence after (9.8)
Replace $f$ by $g$.

p. 175: Exercise 9.3
In (a), the constant $c$ actually depends on $f$ and $n$ but not on $R$. The constant $C$ in (b) depends on $n$ as well as $R$. 

Chapter 8

Chapter 9

Chapter 10
p. 178: Equation (10.4)
The dimension is meant to be $n$ here:
\[
\int_{\Omega} u \frac{\partial \psi}{\partial x_j} \, d^n x = - \int_{\Omega} f \psi \, d^n x
\]
for all $\psi \in C^\infty_{\text{cpt}}(\mathbb{R}^n)$.

p. 179: Lemma 10.1
This proof is incorrect: $L^1_{\text{loc}}$ is not contained in $L^2_{\text{loc}}$. This Lemma requires some basic measure theory arguments, which I was trying to avoid since measure theory is not assumed as a prerequisite. The standard proof would be to introduce a function $\psi \in C^\infty_{\text{cpt}}(\mathbb{R}^n)$ with
\[
\int_{\mathbb{R}^n} \psi \, d^n x = 1.
\]
For $\delta > 0$, defined the rescaled function $\psi_\delta(x) := \delta^{-n} \psi(x/\delta)$. Then $f * \psi_\delta = 0$ for all $\delta > 0$, by hypothesis. A ‘mollification’ argument shows that $f * \psi_\delta \to f$ as $\delta \to 0$, both in $L^1_{\text{loc}}$ and also pointwise almost everywhere.

p. 188: Equation (10.18)
The definition should have $m = 1$;
\[
H^1_0(\Omega) = \left\{ u \in H^1(\Omega); \lim_{k \to \infty} \| u - \psi_k \|_{H^1} = 0 \text{ for } \psi_k \in C^\infty_{\text{cpt}}(\Omega) \right\}.
\]

p. 192: Theorem 10.13
The dimension should be $n$: “A function $f \in L^2(\mathbb{T}^n)$ lies in $H^m(\mathbb{T}^n)$ for . . .” The $2\pi$ in the second line of the final equation should be $(2\pi)^n$.

Acknowledgments: Thanks for Harald Hanche-Olsen, Chris Kottke, and Wei Hu for pointing out corrections to the text.