Errata for Spectral Theory

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Chapter 4

p. 85: Second resolvent identity
This formula should read,
\[(S - z)^{-1} - (T - z)^{-1} = (S - z)^{-1}(T - S)(T - z)^{-1},\]
both here and in Exercise 4.1

p. 96: Exercise 4.1
The second resolvent identity should read
\[(S - z)^{-1} - (T - z)^{-1} = (S - z)^{-1}(T - S)(T - z)^{-1}.\]

p. 116: Proof of Thm 5.11
Missing brackets in the second paragraph. The function $Q^{-1}\phi$ has support on \(\{\alpha = \lambda\}\).

p. 133: First equation
The $\gamma$ is erroneous. The first equation should read
\[|\langle f, v \rangle| \leq \|f\|\|v\| \leq \|f\|\|v\|_{H^1},\]

p. 136: Proof of Thm 6.8
The first sentence of the second paragraph is mixed up. This paragraph should read as follows:
If $u \in \mathcal{D}(-\Delta_D)$, then by (6.13) we have
\[(1) \quad \|u\|_{H^1}^2 = \langle u, (-\Delta + 1)u \rangle.\]
By Cauchy-Schwarz and the fact that $\|u\| \leq \|u\|_{H^1}$, this implies that
\[\|u\|_{H^1} \leq \|(-\Delta + 1)u\|.\]
This shows that $(-\Delta_D + 1)^{-1}$ is bounded as a map $L^2(\Omega) \to H^1_0(\Omega)$. Therefore $(-\Delta_D + 1)^{-1}$ is compact as a map $L^2(\Omega) \to L^2(\Omega)$ by Theorem 6.9.

p. 144: Corollary 6.16
The assumption on $\psi$ should read $\psi \in H^1(\Omega)$.

p. 151: Equation (6.48)
The $\sigma$ in the brackets should be $\sigma(-\Delta_D)$. 

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p. 151: Equation (6.49)  
These inequalities are backwards. The equation should read:

\[ N_{R_1}(t) \leq N_{\Omega}(t) \leq N_{R_2}(t). \]

p. 172: Proof of Thm 6.34  
The definition of \( \psi_1^\pm \) should read

\[ \psi_1^\pm(x) := \max\{\pm \psi_1(x), 0\}. \]

p. 195: Proof of Thm 7.7  
In last part of the proof, \( B \) was mistakenly assumed to be closed. Here is a clean version of the final two paragraphs:

Now assume that \( A \) is merely essential self-adjoint. If \( u \in \mathcal{D}(A) \), then there exists a sequence \( u_n \to u \) with \( u_n \in \mathcal{D}(A) \), such that \( Au_n \) converges to \( Au \). By the assumption (7.24), the sequence \( Bu_n \) also converges, so that \( u \in \mathcal{D}(B) \) (since \( B \) is closed). By continuity, we can extend (7.24) to

\[ \|B u\| \leq \alpha \|Au\| + \beta \|u\| \]

for all \( u \in \mathcal{D}(A) \). By the first part of the proof, this implies that \( \overline{A} + \overline{B} \) is self-adjoint on the domain \( \mathcal{D}(\overline{A}) \).

It remains to check that \( \overline{A + B} = \overline{A} + \overline{B} \). Since \( \overline{A + B} \) is a closed extension of \( A + B \), we have \( \overline{A + B} \subset \overline{A} + \overline{B} \). On the other hand, the assumption (7.24) gives

\[ \|(A + B)u\| \leq (\alpha + 1) \|Au\| + \beta \|u\|. \]

For \( u \in \mathcal{D}(A) \) this implies that \( u \in \mathcal{D}(A + B) \) and that \( (A + B)u = (\overline{A} + \overline{B})u \). In other words,

\[ \overline{A + B} \subset \overline{A} + \overline{B}. \]

We conclude that \( \overline{A + B} \) is self-adjoint on \( \mathcal{D}(A) \).