1. We want to prove, by induction, that, for every positive integer $n$,
   \[ 1 + 2 + 3 + \cdots + n = \frac{n(n + 1)}{2}. \]
   a) What is the open statement “$P(n)$”?
   
   \[ P(n) = \]

   b) What is the statement “$P(1)$”? Why is $P(1)$ true?
   
   \[ P(1) = \]

   c) What is the inductive step? Write out your assumption, your desired conclusion, and the inductive step (i.e., the proof that $P(n - 1) \Rightarrow P(n)$).
   Assume that

   We want to show that

   (Inductive step)

2. Let $a_n$ be a sequence such that $a_1 = 1$ and $a_n = na_{n-1}$. We want to prove, by induction, that, for every positive integer $n$,
   \[ a_n = n! = n(n - 1)(n - 2) \cdots 2 \cdot 1. \]
   a) What is the open statement “$P(n)$”?
   
   \[ P(n) = \]

   b) What is the statement “$P(1)$”? Why is $P(1)$ true?
   
   \[ P(1) = \]

   c) What is the inductive step? Write out your assumption, your desired conclusion, and the inductive step (i.e., the proof that $P(n - 1) \Rightarrow P(n)$).
   Assume that
We want to show that

(Inductive step)

3. We want to prove, by induction, that, for every positive integer \( n \),
\[
1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n + 1)^2}{4}.
\]
a) What is the open statement “\( P(n) \)”?
\[ P(n) = \]

b) What is the statement “\( P(1) \)”？ Why is \( P(1) \) true?
\[ P(1) = \]

c) What is the inductive step? Write out your assumption, your desired conclusion, and the inductive step (i.e., the proof that \( P(n - 1) \Rightarrow P(n) \)).

Assume that

We want to show that

(Inductive step)

4. Prove, by induction, that \( 2^{n+1} \geq n^2 \) for every integer \( n \). (For this problem, you will have to first check \( P(1) \) and \( P(2) \).)