(1) Which of these are binary operations? Which are commutative/associative, which have an identity element, and for which does every element have an inverse?

(a) \( \mathbb{Z} \times \mathbb{Z} \overset{+}{\rightarrow} \mathbb{Z} \).
(b) \( \mathbb{R} \times \mathbb{R} \overset{\cdot}{\rightarrow} \mathbb{R} \).
(c) \( \mathbb{R} \times \mathbb{R} \overset{-}{\rightarrow} \mathbb{R} \).
(d) \( \mathbb{R}^* \times \mathbb{R}^* \overset{\cdot}{\rightarrow} \mathbb{R}^* \) (where \( \mathbb{R}^* = \mathbb{R} - \{0\} \)).
(e) \( \text{Fun}(B, B) \times \text{Fun}(B, B) \overset{\circ}{\rightarrow} \text{Fun}(B, B) \).
(f) \( \mathbb{Z} \times \mathbb{Z} \overset{\ast}{\rightarrow} \mathbb{Z} \) (where \( a \ast b = a + b + 1 \)).
(g) \( \mathbb{Z} \times \mathbb{Z} \overset{\star}{\rightarrow} \mathbb{Z} \) (where \( a \star b = 2a + b \)).
(h) \( P(A) \times P(A) \overset{\cup}{\rightarrow} P(A) \).
(i) \( P(A) \times P(A) \overset{\cap}{\rightarrow} P(A) \).
(j) \( \mathbb{R} \cup \{\infty\} \times \mathbb{R} \cup \{\infty\} \overset{\oplus}{\rightarrow} \mathbb{R} \cup \{\infty\} \) (where \( a \oplus b = \max(a, b) \)).
(k) \( \mathbb{R} \cup \{\infty\} \times \mathbb{R} \cup \{\infty\} \overset{\ominus}{\rightarrow} \mathbb{R} \cup \{\infty\} \) (where \( a \ominus b = \min(a, b) \)).
(l) \( \{0, 1\} \times \{0, 1\} \overset{\star}{\rightarrow} \{0, 1\} \) (where \( 0 \star 0 = 0, 0 \star 1 = 1, 1 \star 0 = 0, 1 \star 1 = 0 \)).

Answers (please circle):

(a) commutative associative identity inverses
(b) commutative associative identity inverses
(c) commutative associative identity inverses
(d) commutative associative identity inverses
(e) commutative associative identity inverses
(f) commutative associative identity inverses
(g) commutative associative identity inverses
(h) commutative associative identity inverse
(i) commutative associative identity inverse
(j) commutative associative identity inverse
(k) commutative associative identity inverses

(2) Try to think of 3 more examples of binary operations.